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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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No. 911  
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SOME INVESTIGATIONS OF THE GENERAL INSTABILITY  
OF STIFFENED METAL CYLINDERS

VII - STIFFENED METAL CYLINDERS SUBJECTED  
TO COMBINED BENDING AND TORSION

Guggenheim Aeronautical Laboratory  
California Institute of Technology

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Washington  
November 1943







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This is the seventh of a series of reports covering an investigation of the general instability problem by the California Institute of Technology. The first five reports of this series cover investigations of the general instability problem under the loading conditions of pure bending and were prepared under the sponsorship of the Civil Aeronautics Administration. This report and succeeding reports of this series cover the work done on other loading conditions under the sponsorship of the National Advisory Committee for Aeronautics.

SUMMARY

This report summarizes the work that has been carried on in the experimental investigation of the problem of the general instability of stiffened metal cylinders subjected to combined bending and torsion at the C.I.T. This part of the investigation included tests on 26 sheet-covered specimens. An interaction curve for the case of combined bending and torsion is presented. The results of tests of 17 specimens subjected to pure torsion are also given.

INTRODUCTION

It is intended to give in this report a summary of the experimental investigation of the general instability



of stiffened metal cylinders subjected to combined bending and torsion. The earlier work at the C.I.T. on the problem of the general instability of stiffened metal cylinders has been reported in references 1, 2, 3, 4, 5, and 6. A total of 26 sheet-covered specimens has been tested to determine the effects of combined loading and bending plus torsion, and an additional 17 specimens have been tested to determine the effects of a pure torsion load. In the present report, the results of tests in combined bending and torsion are discussed and the test data of the pure torsion tests are also included.

## TESTS IN COMBINED BENDING AND TORSION

### Testing Apparatus

A complete description of the testing machine used in these tests is given in reference 2. This machine (fig. 1) is adapted to the application of either pure bending or pure torsion or any combination of bending and torsion.

For the present tests, it was desired to apply a uniform bending and uniform torsion simultaneously. The ratio of the two loads was different for different specimens. In order to apply uniform bending, the moment arms were connected by an extra flexible aircraft cable attached at one end to a loading screw which in turn was attached to a loading dynamometer. For the application of uniform torsion, one of the torsion arms was fixed while the load was applied to the arm by means of a loading screw.

### Testing Procedure

The desired loading required a simultaneous application of bending and torsion, the ratio of the two loads being fixed by any given test. During the application of any load increment the ratio of bending to torsion was closely maintained. This result was accomplished by preparing a loading schedule for each test and, by calling out the bending load as it was applied, the two operators could maintain the proper loading ratio.

The axial strain in the longitudinal stiffeners was measured by means of resistance strain gages of 16-inch



length. These strain gages were mounted on the neutral axis of the stiffeners and were also mounted in pairs - one on each side of the stiffener - in order that the mean reading would be unaffected by bending. These strain measurements were taken at the center of the specimen, that is, the center of the strain gage was placed at the center of the cylinder.

Inasmuch as the specimen was subjected to uniform loadings, both bending and torsion, the unit axial strain for any loading increment can also be obtained by measuring the mean strain over the entire specimen. The measurements were obtained by measuring the over-all deformation of the specimen by means of a dial gage mounted on a rigid rod. A sharp pointed fitting was attached to one end of the rod and another to the dial plunger. Inserting these pointed fittings into punch marks on the mounting ring near the surface of the specimen allowed consistent measurements of the over-all deformation to be obtained for each load increment.

In order to investigate the complete range of the ratios of bending to torsion, a specimen was first tested under pure torsion and the ratio of bending to shear was then increased in subsequent tests. The ultimate strength in pure bending was obtained from the experimental curve of the pure bending tests given in reference 5.

#### Results of Combined Loading

For the investigation of combined loadings only three types of cylinders were tested, namely, two series that had a frame spacing of 4 inches and a longitudinal stiffness spacing of 5.06 and 2.53 inches in which the frames were  $F_5$  and the longitudinals  $S_2$ ; and a series that had a frame spacing and longitudinal stiffness spacing of 4 and 2.53 inches, respectively, in which the frames were  $F_5$  and the longitudinals  $S_1$ . The frame and longitudinal cross sections are shown in figure 2 and the positions of the dial strain-gages is indicated in figure 3. It was felt that three types of cylinders would be sufficient to determine the dependence of the failing load on the ratio of the combined loading. The experimental data for all test specimens are given in table I.

The compressive unit strain for the longitudinal farthest from the neutral axis as a function of the applied



bending moment is shown for the various tests in figures 4 to 28. The resulting unit strain as obtained from the resistance strain gages and the unit strain obtained from the dial-gage measurements are indicated in each figure. These results indicate that, in general, fair agreement is obtained between these two types of measurements. In only a few tests is the agreement exact.

The variation of the stiffener strain around the circumference as a function of the applied load is also shown for a number of specimens (figs. 4, 5, 7, 8, 11, 12, 15, and 17). The circumferential position of the various gages is indicated in figure 3. The results of these tests have been cross-plotted in figures 29 to 35 to indicate the distribution of normal stiffener strain for increasing loads as a function of the distance from the horizontal axis. The results of figure 29 indicate that the torsional load results in a secondary compressive strain of nearly constant magnitude.

Under a condition of pure bending, a shift in the neutral axes toward the tension side occurs when compression buckling of the sheet takes place. The secondary compressive strain causes a farther shift. In particular, when the ratio of bending to torsion is small, the distribution of strain will be compressive in nature over the entire specimen (fig. 30). This secondary compression strain is probably entirely due to the diagonal tension field that develops in the sheet covering.

The ultimate stress at failure has been presented in the form of the well-known interaction curves; that is,  $\sigma/\sigma_0$  is plotted as a function of  $\tau/\tau_0$  where  $\sigma$  is the normal compression stress at failure for combined loading and  $\sigma_0$  is the same stress at failure for pure bending, and similarly,  $\tau$  and  $\tau_0$  are the shearing stresses at failure for combined loading and pure torsion, respectively. The shearing stresses are computed by the equation

$$\tau = \frac{M}{2 A t}$$

where

M applied torque, pounds



A      area enclosed by the sheet covering, square inches  
t      sheet thickness, inches

Two sets of data are shown. The first set (fig. 36) corresponds to stresses as measured by the resistance strain gages and the second set (fig. 37) to the stresses as obtained from the dial-gage measurements. In either case a considerable amount of scatter is evident. The interaction equations of the form

$$\left(\frac{\sigma}{\sigma_0}\right)^2 + \left(\frac{\tau}{\tau_0}\right)^2 = 1 \quad (1)$$

and

$$\frac{\sigma}{\sigma_0} + \left(\frac{\tau}{\tau_0}\right)^2 = 1 \quad (2)$$

are compared with the experimental results. Figure 36 indicates that for the resistance strain-gage measurements about two-thirds of the test values lie fairly close to the curve of equation (2); whereas the majority of the dial-gage measurements scatter about the curve of equation (1). It should be noted that for the pure bending specimen (specimen 159), the strain at failure measured by the resistance strain gages is considerably lower than the strain of the preceding combined loading specimens. For this reason a failing strain of 0.0019 was used to determine  $\sigma_0$ . This value was obtained from the pure bending parameter and agrees fairly well with the dial gage reading.

#### PURE TORSION

So far, the investigation has covered the ultimate strength of reinforced cylinders under pure bending and the influence of combined loadings; that is, combined bending and transverse shear and combined bending and torsion. In order to use this data for design purposes, it is still necessary to determine the ultimate strength of reinforced cylinders when loaded in pure shear, that is, a pure torsion loading. Tests of this nature are



necessary in order to ascertain the value of  $\tau/\tau_0$  for any given design problem.

An attempt will be made to determine a suitable shear parameter in a manner similar to that used to determine the pure bending parameter. This method requires a systematic investigation of the variables dealing with the geometry of the structure and the variables that involve the sectional properties of the stiffening elements as well as the sheet covering. Seventeen pure torsion tests have been completed and the specimens and test data for these tests are given in table II. Additional testing is necessary before an analysis of the data can be attempted. Photographs (figs. 38 to 44) show a number of specimens after failure. For purposes of comparison, photographs are included of specimens in combined bending and torsion.

#### CONCLUSIONS

(1) Inasmuch as the specimens were built and tested with the usual care of experimental work, it is felt that similar structures built under production conditions would be subject to at least the same variations of ultimate strength as the specimens that were tested. The experimental results of the tests in combined bending and torsion are therefore considered satisfactory in that they indicate the lower strength limit which can be expected in practice. It is seen from figures 2 and 3 that equation (2) described this lower limit fairly well.

(2) Additional testing is necessary before an analysis of the data on pure torsion can be attempted.

Guggenheim Aeronautical Laboratory,  
California Institute of Technology,  
Pasadena, Calif., Feb. 18, 1943.



## REFERENCES

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TABLE I

## COMBINED BENDING AND TORSION

Specimen	Sheet thickness (in.)	Radius (in.)	Frame	Stiffener	Frame spacing (in.)	Longitudinal spacing (in.)	Torsion bending	Ultimate bending moment (in.-lb)	Ultimate torsional moment (in.-lb)	Compressive strain at failure (in./in.)	Shear stress at failure (lb/sq in.)
149	.010	16	$F_s$	$S_2$	4	5.06	Torsion	0	$1070 \times 10^2$	$5.60 \times 10^{-4}$	6650
150	↑	↑	↑	↑	↑	↑	5/1	$216 \times 10^2$	1080	7.64	6710
151	↑	↑	↑	↑	↑	↑	3/1	287	860	7.12	5340
152	↑	↑	↑	↑	↑	↑	2/1	472	945	10.70	5870
153	↑	↑	↑	↑	↑	↑	1/1	770	770	13.5	4780
154	↑	↑	↑	↑	↑	↑	1/1.5	900	600	14.6	3730
155	↑	↑	↑	↑	↑	↑	1/2	1000	500	18.6	3110
156	↑	↑	↑	↑	↑	↑	1/2.5	1175	470	18.05	2920
157	↑	↑	↑	↑	↑	↑	1/3	1150	385	16.2	2390
158	↑	↑	↑	↑	↑	↑	1/3.5	1210	355	16.7	2200
159	↑	↑	↑	↑	↑	↓	Bending	1400	0	16.0	0
160	↑	↑	↑	↑	↑	↓	1/1.5	875	583	14.4	3620
161	↑	↑	↑	↑	↑	↓	1/5	1200	240	16.2	1490
162	↑	↑	↑	↑	↑	2.53	Torsion	0	1300	2.12	8080
163	↑	↑	↑	↑	↑	↑	1/1	820	820	7.00	5100
164	↑	↑	↑	↑	↑	↑	2/1	575	1150	5.27	7150
165	↑	↑	↑	↑	↑	↑	1/2	1390	695	10.95	4320
166	↑	↑	↑	↑	↑	↑	1/3	1900	633	14.00	3930
167	↑	↑	↑	↑	↑	↑	1/5	2540	508	17.6	3160
168	↑	↑	↑	↑	↑	↑	1/2	1600	800	12.5	4970
169	↑	↑	↑	↓	↑	↑	1/1	1050	1050	9.4	6520
170	↑	↑	↑	$S_2$	↑	↑	2/1	550	1100	5.25	6840
171	↑	↑	↑	$S_1$	↑	↑	Torsion	0	1185	1.85	7360
172	↓	↓	↓	↑	↓	↓	1/1	900	900	9.75	5590
173	↓	↓	↓	↑	↓	↓	1/2	1280	645	14.10	4010
174	.010	16	$F_s$	$S_1$	4	2.53	1/4	1880	460	21.40	2860



TABLE II  
PURE TORSION

Specimen	Sheet thickness	Radius (in.)	Frame	Stiffener	Frame spacing (in.)	Stiffener spacing (in.)	Ultimate torsion moment (in.-lb)	Shear stress at failure (lb/sq in.)
175	.010	16	$F_5$	$S_1$	8	2.53	800	4975
176	.010	$\uparrow$			2	2.53	1600	9950
177	.015				16	5.06	500	2070
178	.015				8	$\uparrow$	750	3100
179	.015				4		1090	4520
180	.015				2		1700	7050
181	.020				16		730	2270
182	.020				8	$\downarrow$	925	2870
183	.020				4		1350	4200
184	.020				2	5.06	2335	7240
185	.015	10			8	5.25	380	4030
186	.015	10			4	5.25	630	6680
187	.015	10			2	5.25	890	9440
189	.010	16			32	10.12	130	810
190	.010	$\uparrow$	$F_5$	$S_1$	16	10.12	250	1555
191	.010	$\downarrow$			8	10.12	410	2550
192	.010	16			4	10.12	676	4200



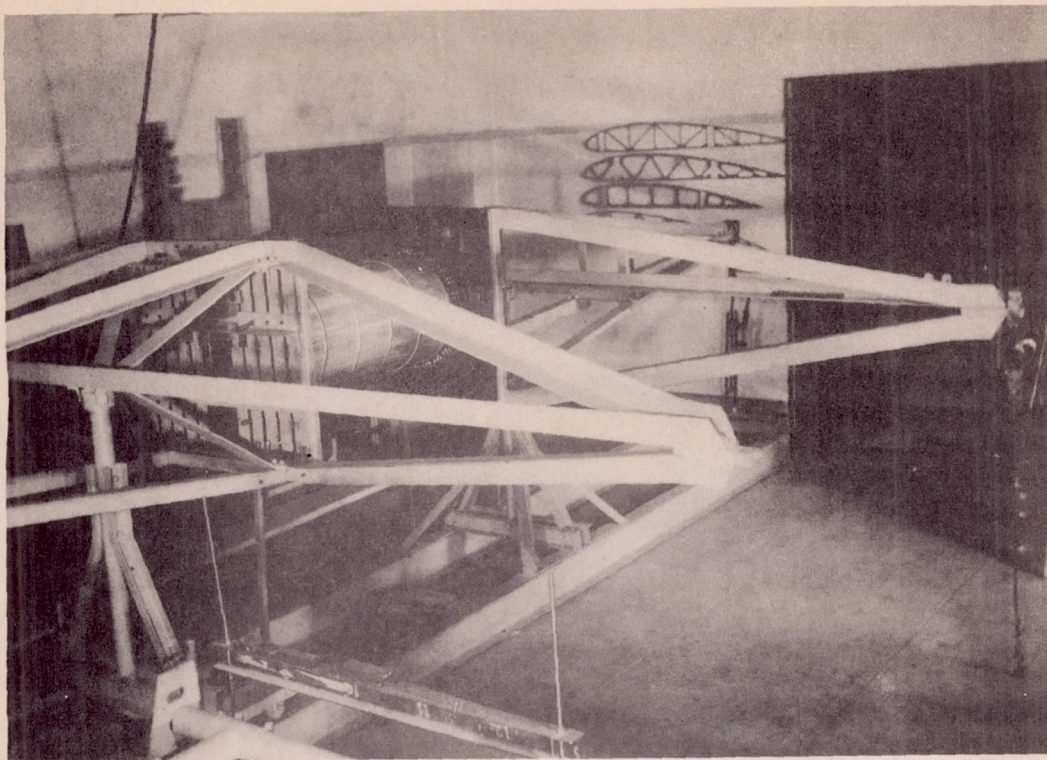


Figure 1.- Testing machine used to apply either pure bending or pure torsion or any combination of bending or torsion.

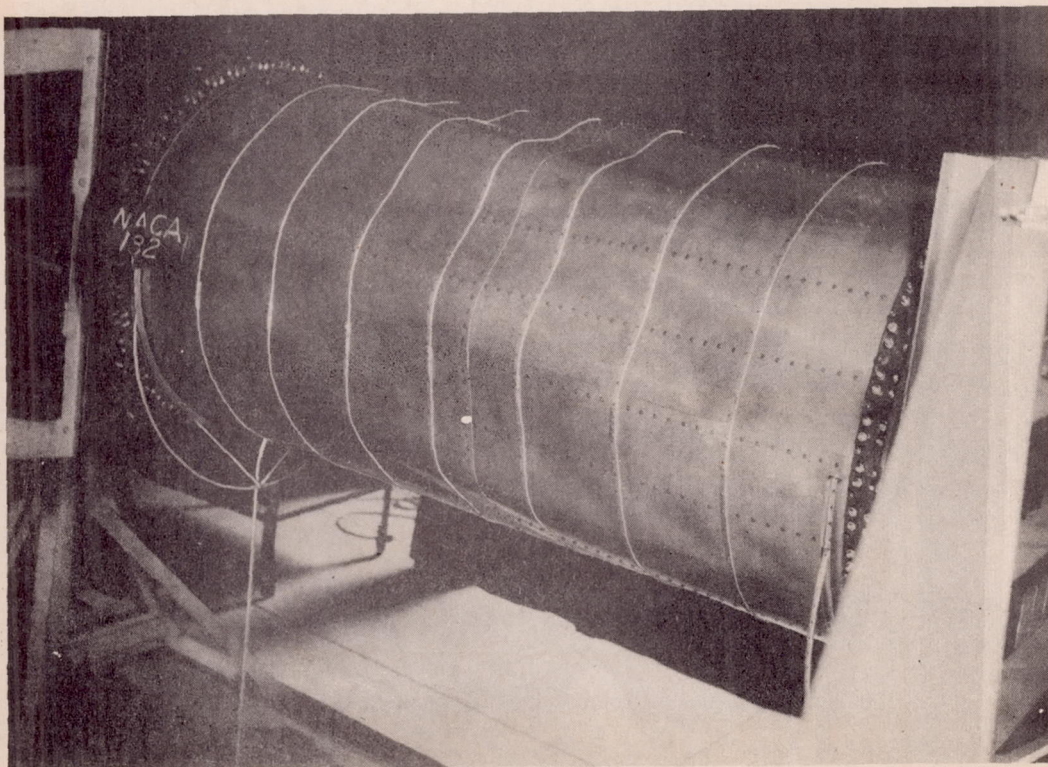
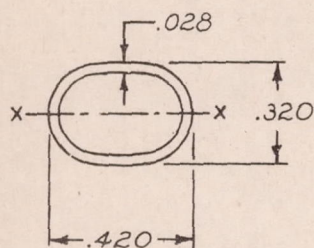


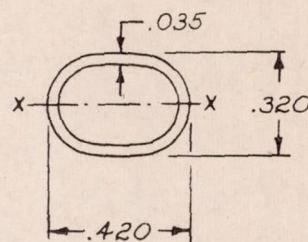
Figure 44.- Pure torsion.





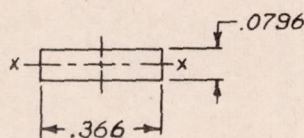
STIFFENER -  $S_1$

AREA = .0324  $I_{xx} = .000374$



STIFFENER -  $S_2$

AREA = .0368  $I_{xx} = .000407$



FRAME -  $F_5$

AREA = .0291  $I_{xx} = 1.537 \times 10^{-5}$

Figure 2.- Frame and longitudinal cross sections used on specimens.



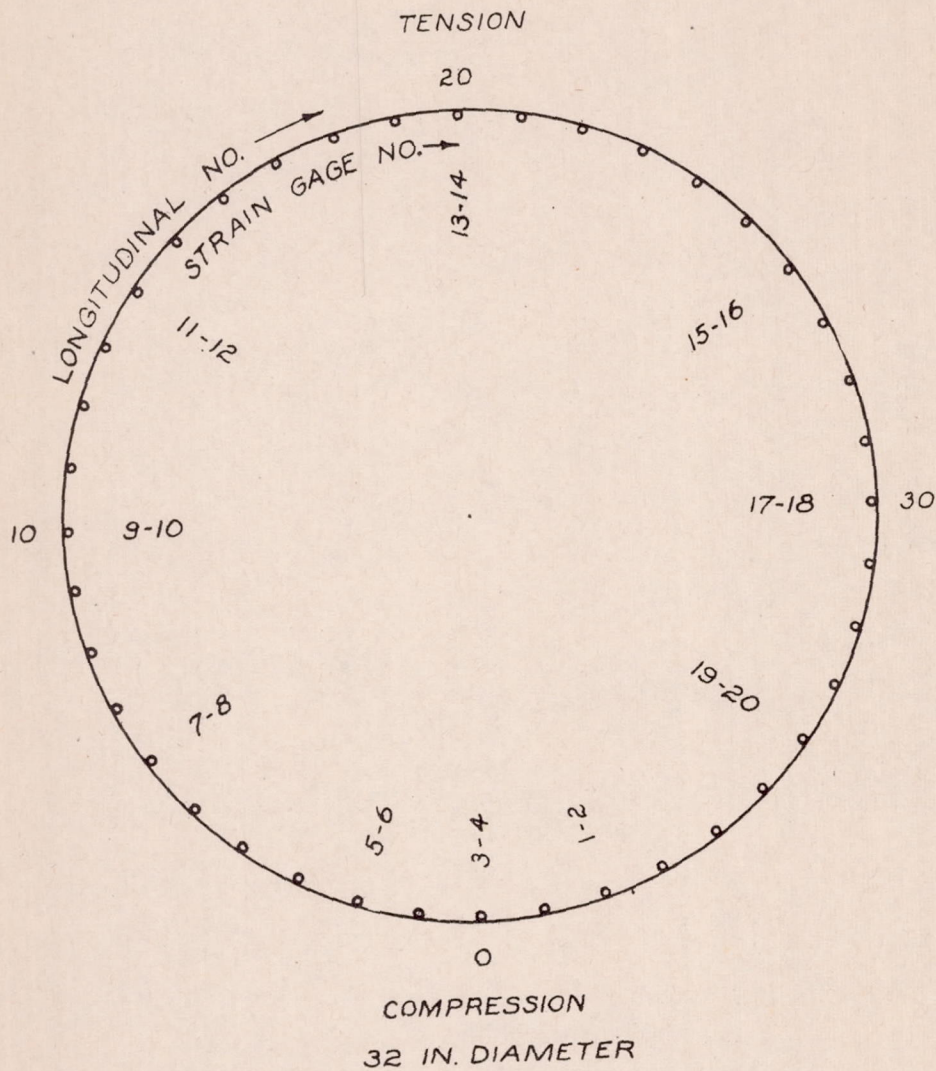
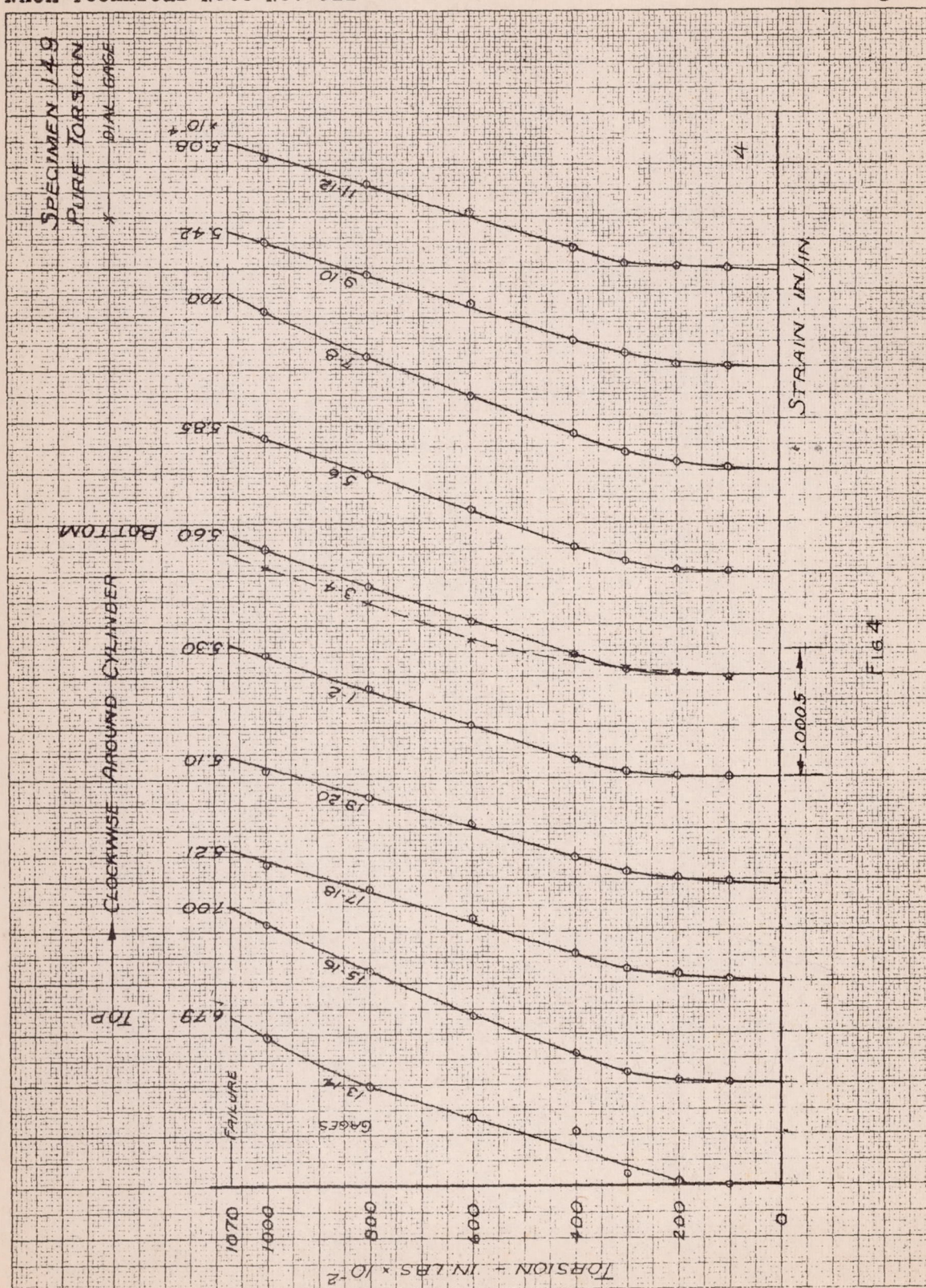
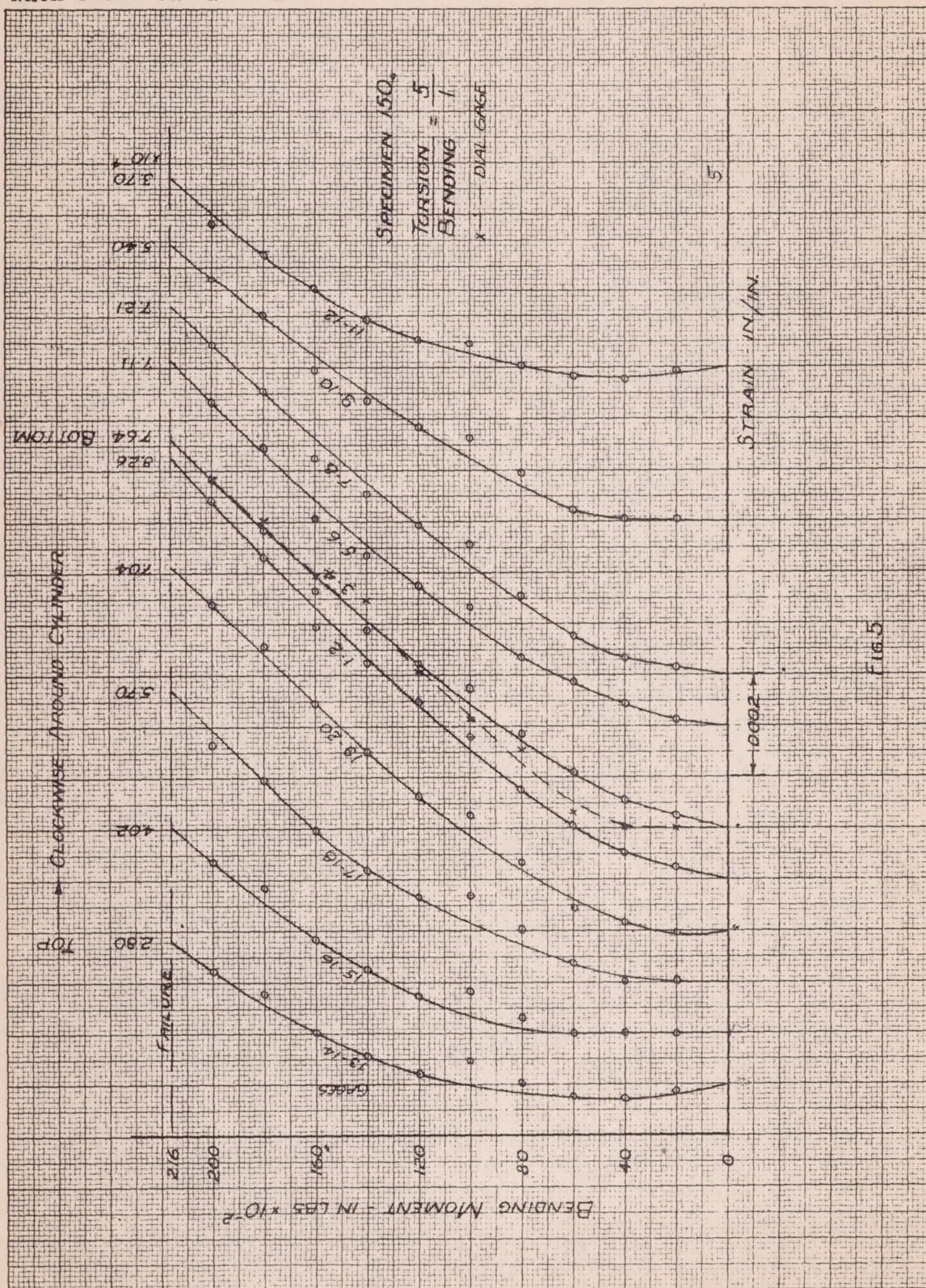


Figure 3.- Cross section of cylinder showing position of the dial strain gages on the circumference.

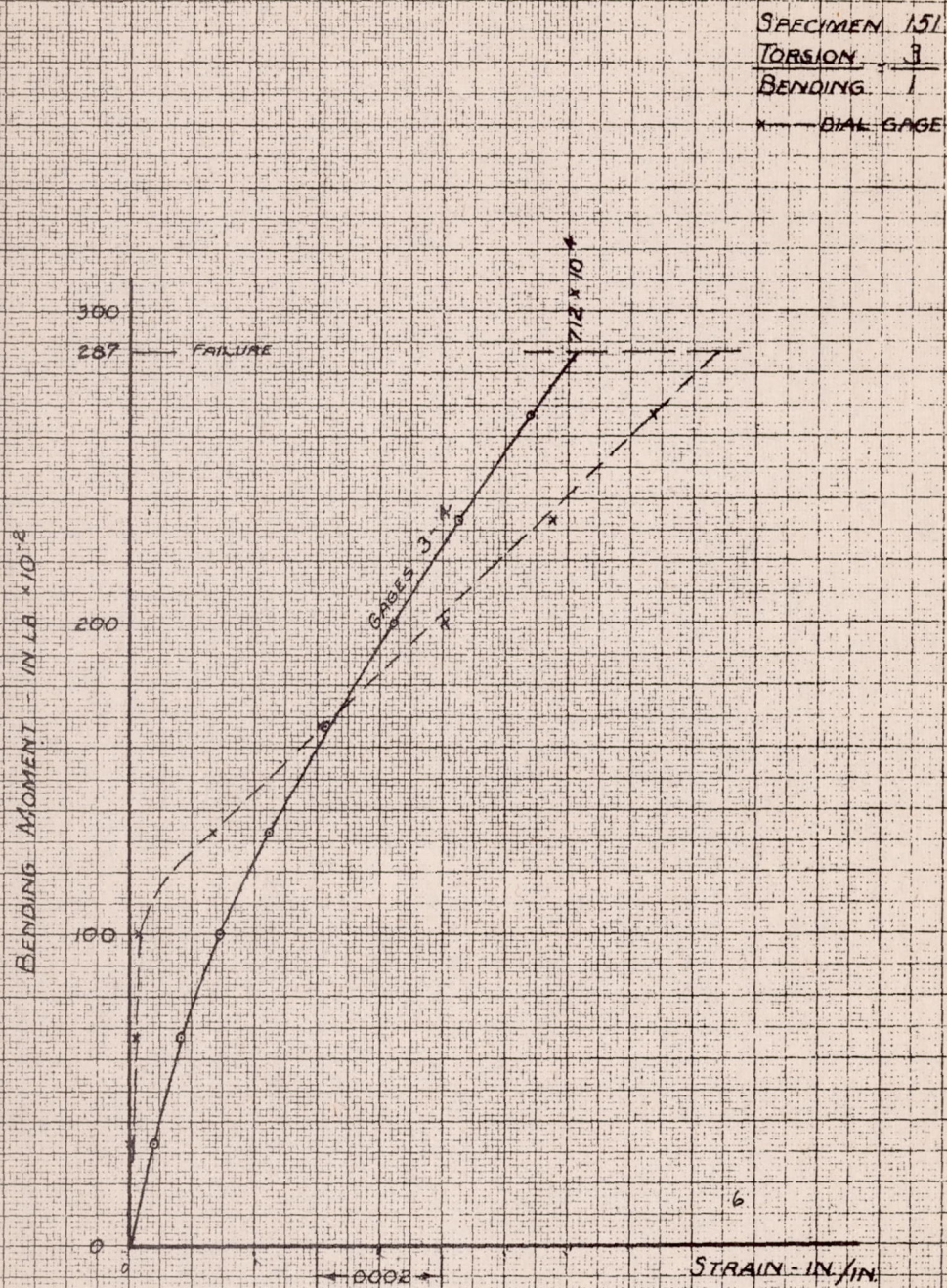














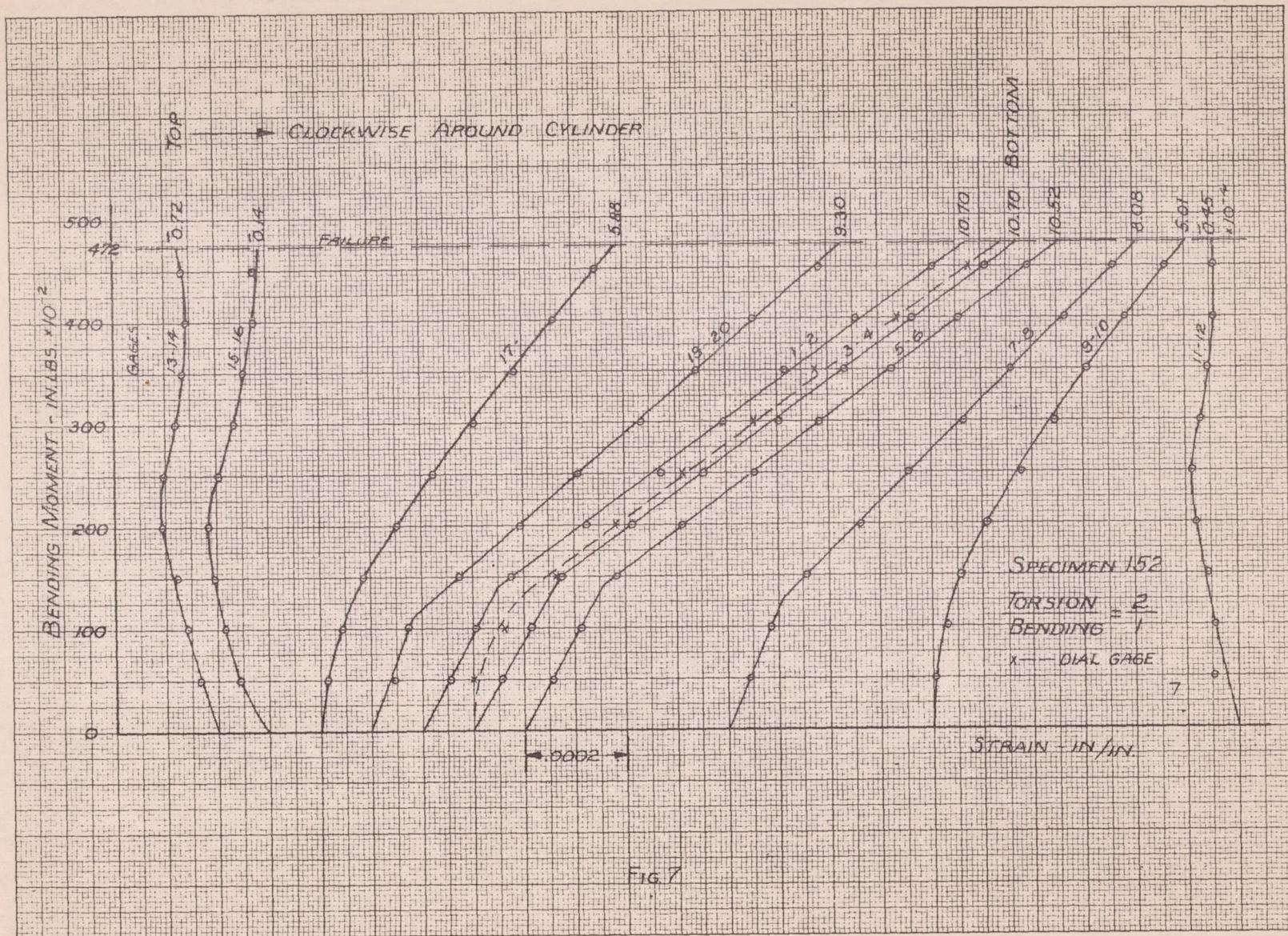
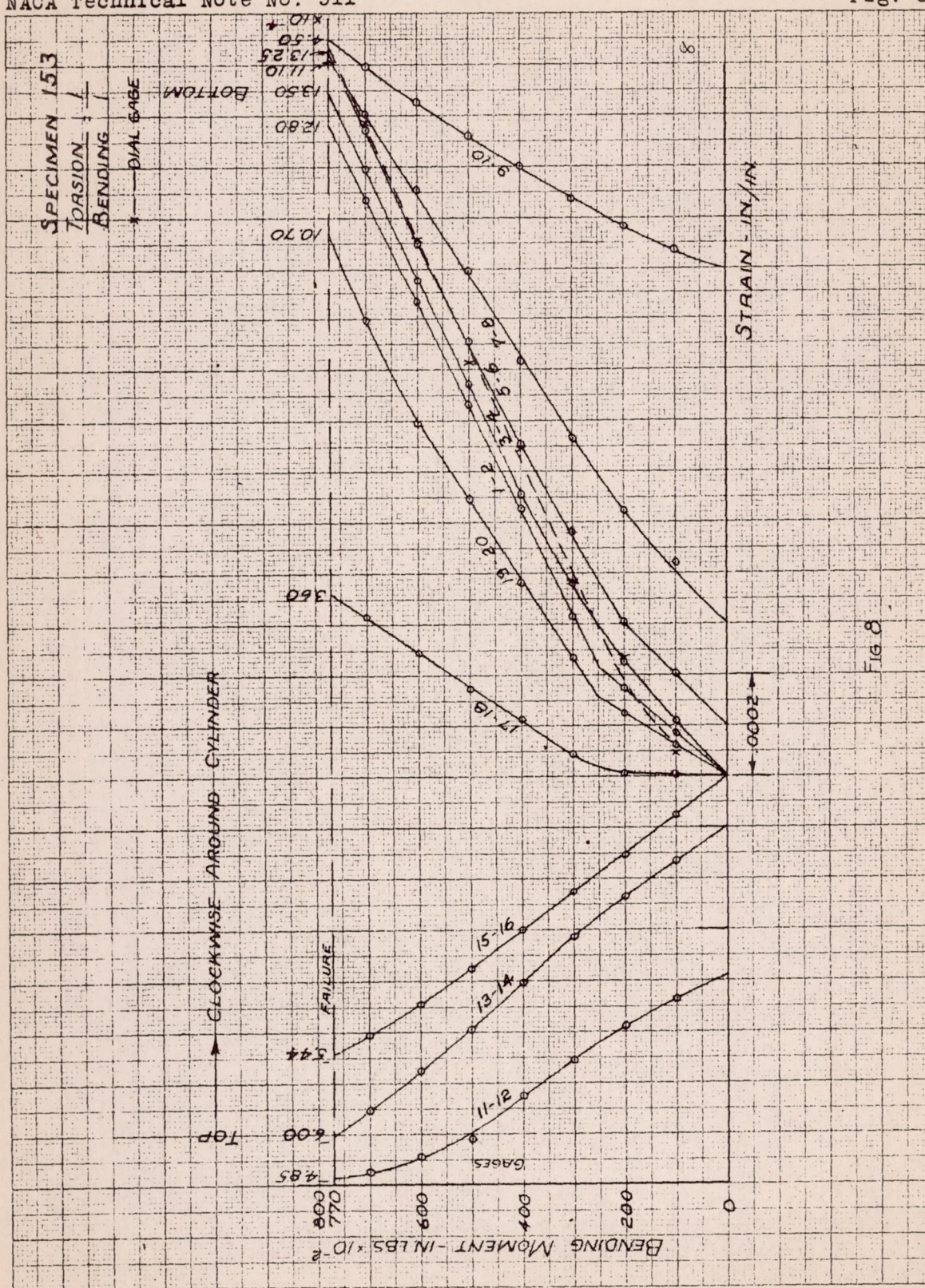
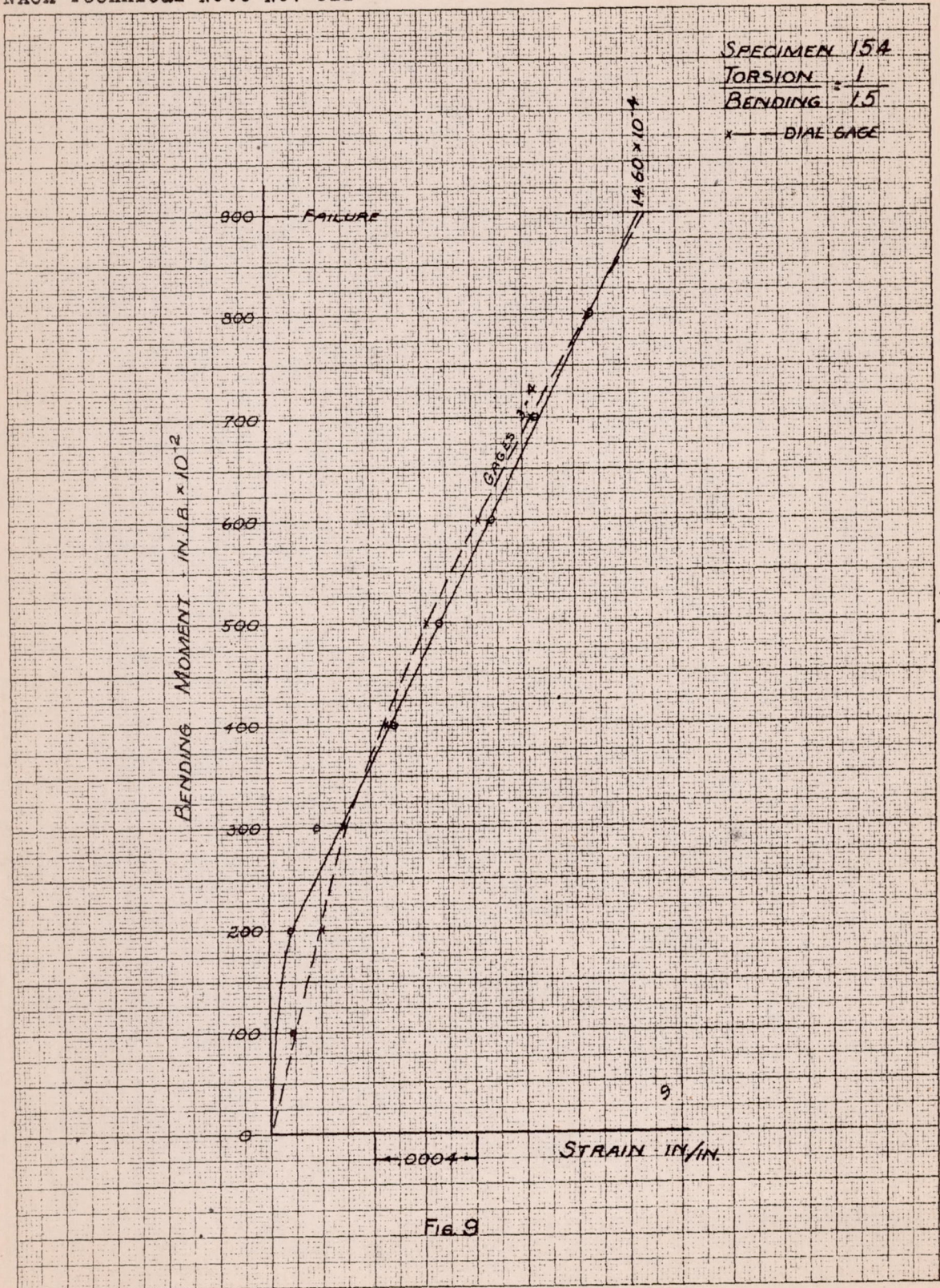


Fig. 7











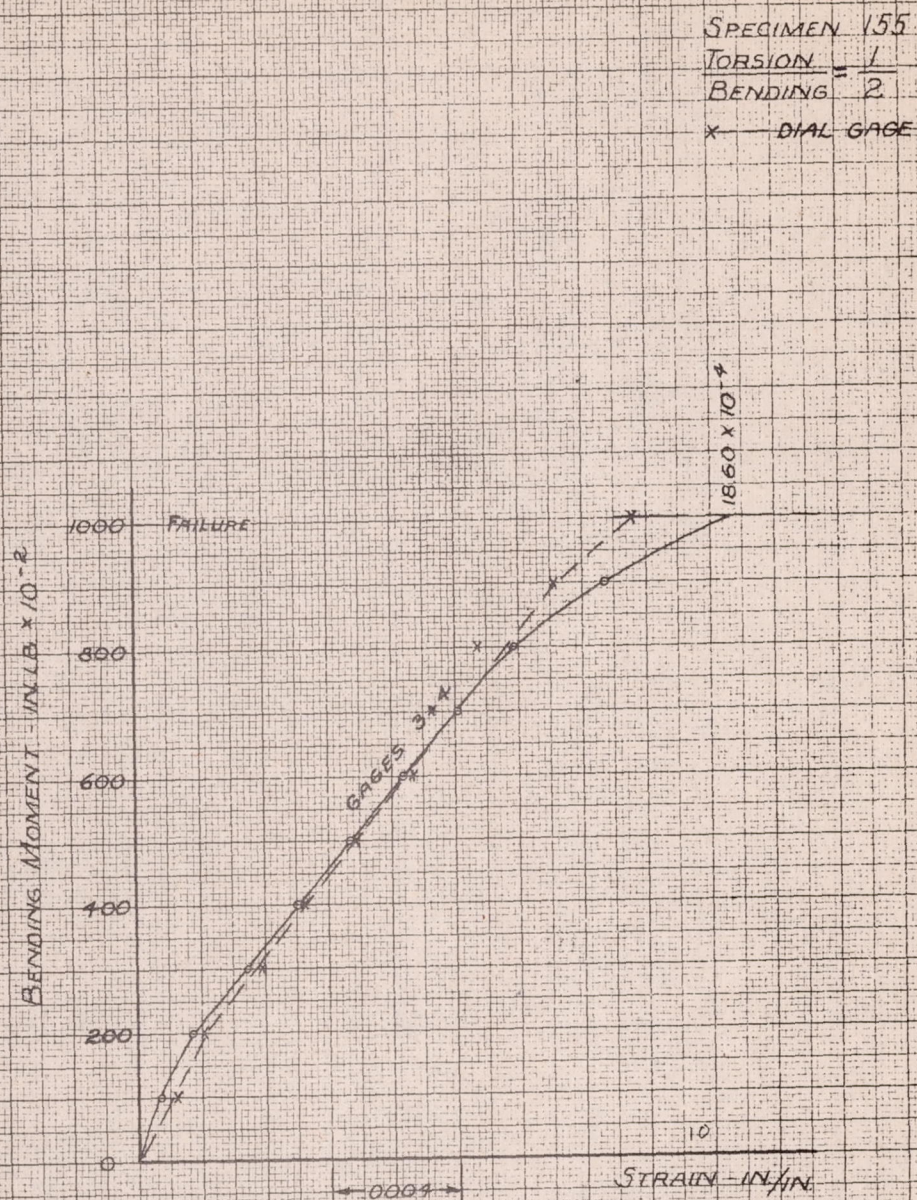


Fig 10



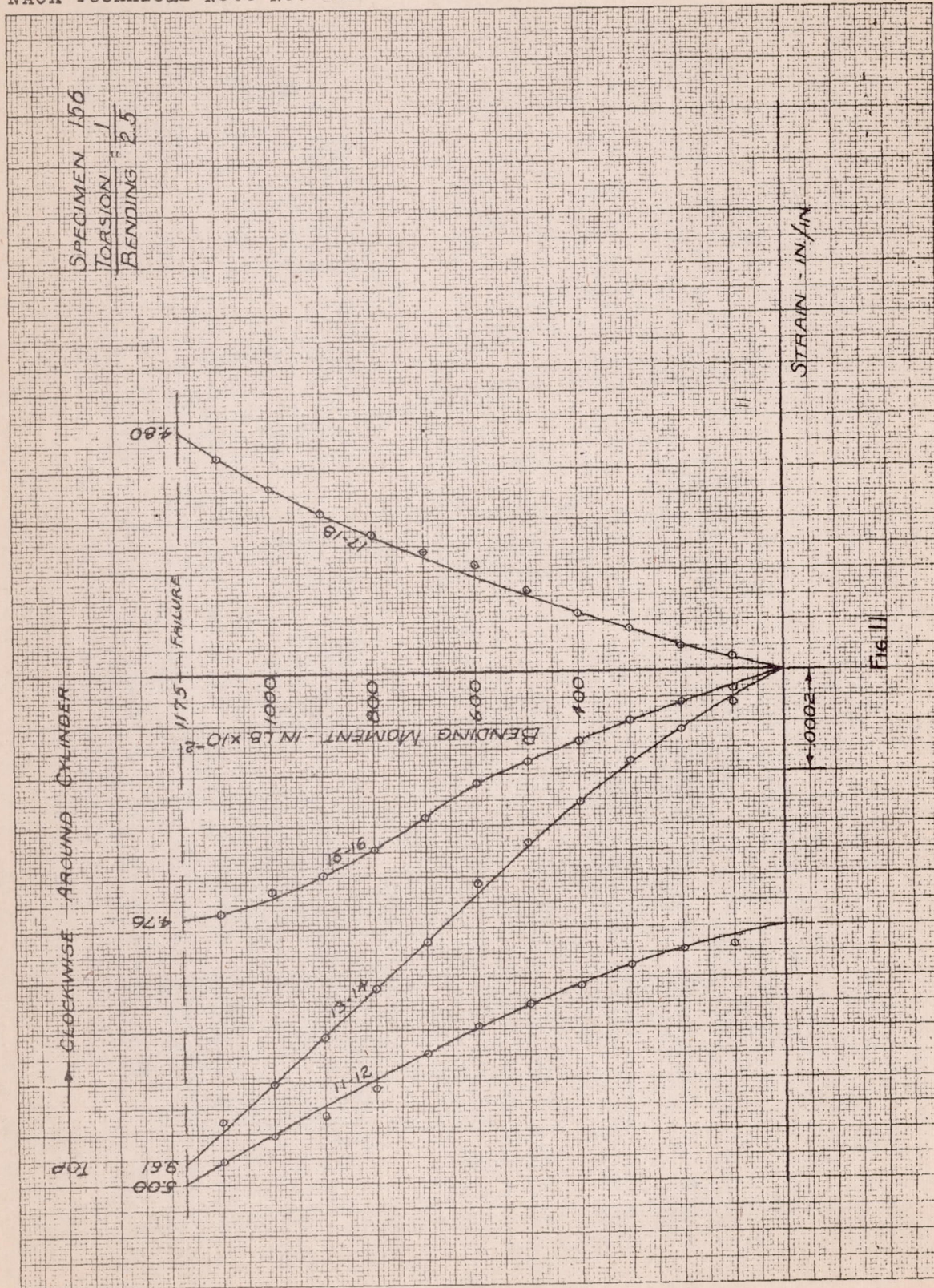
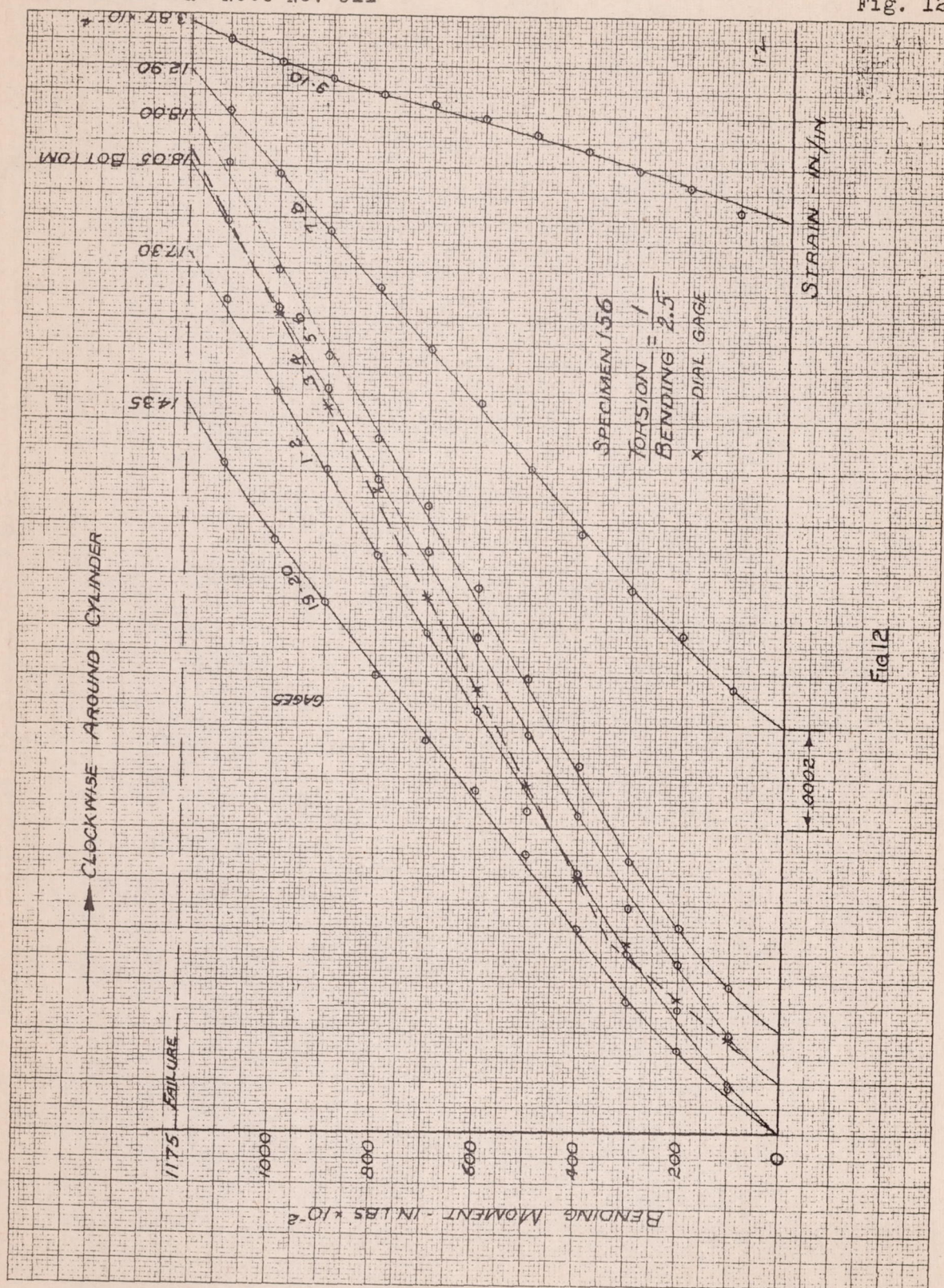
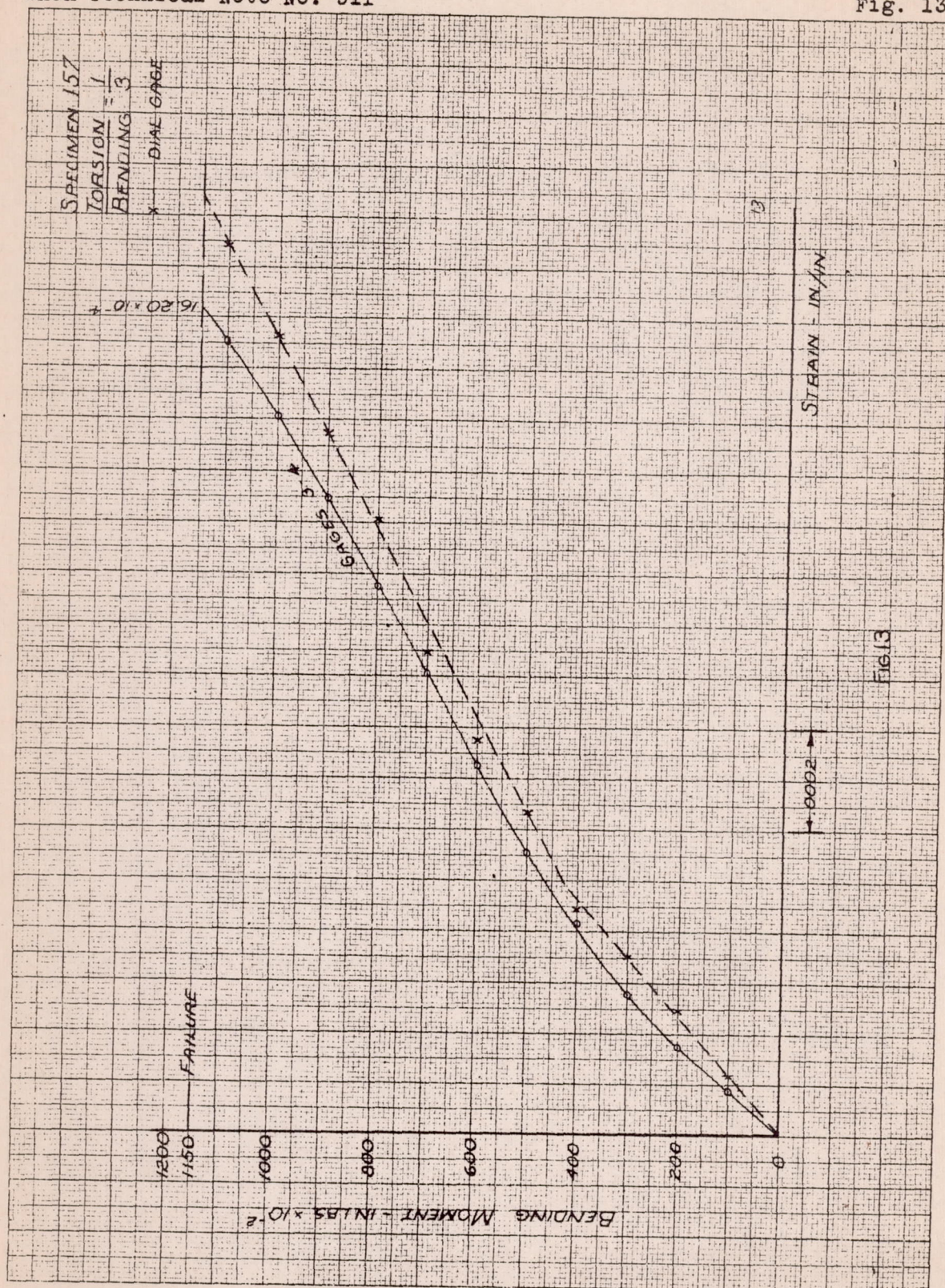




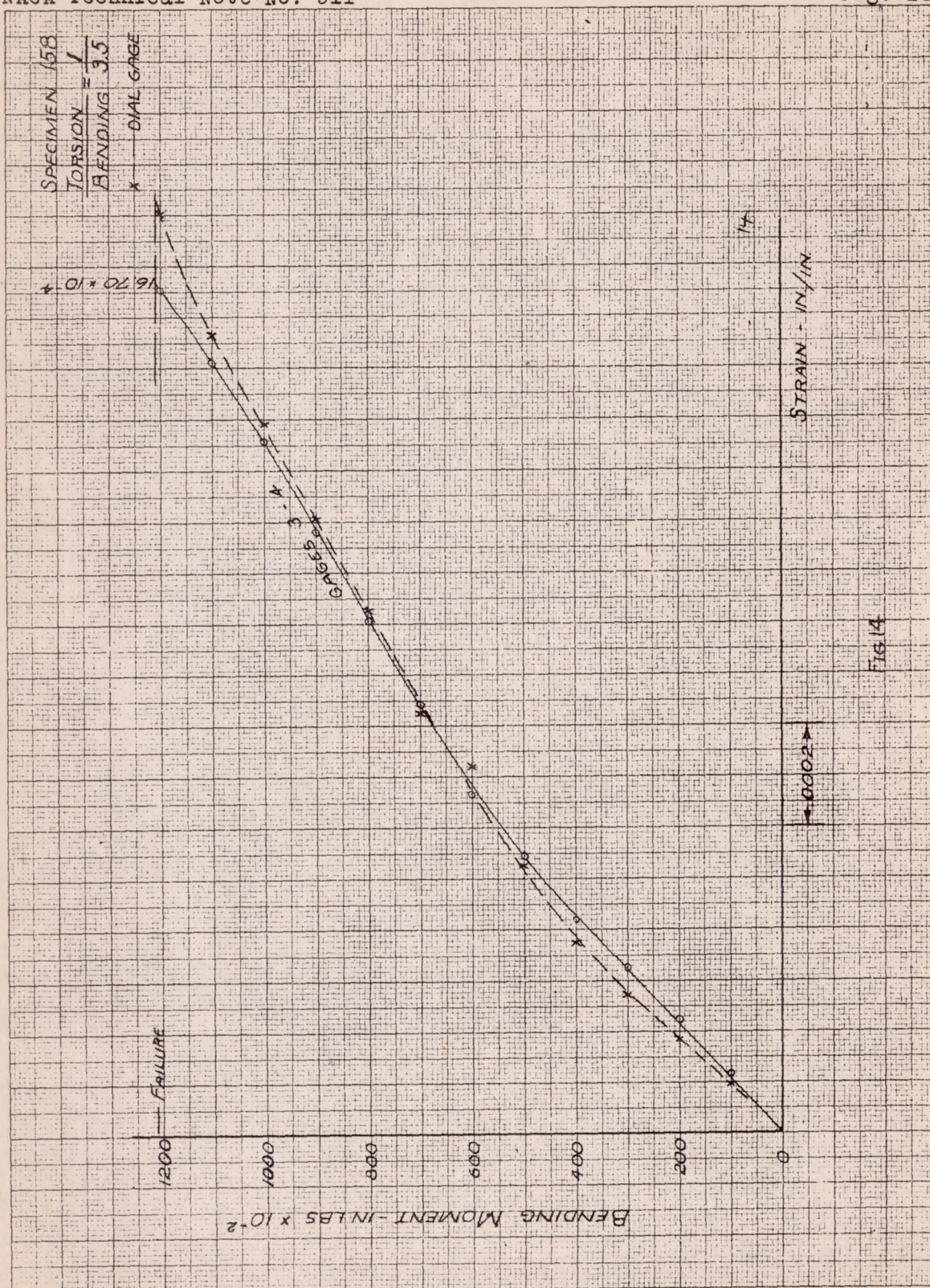
Fig. 12



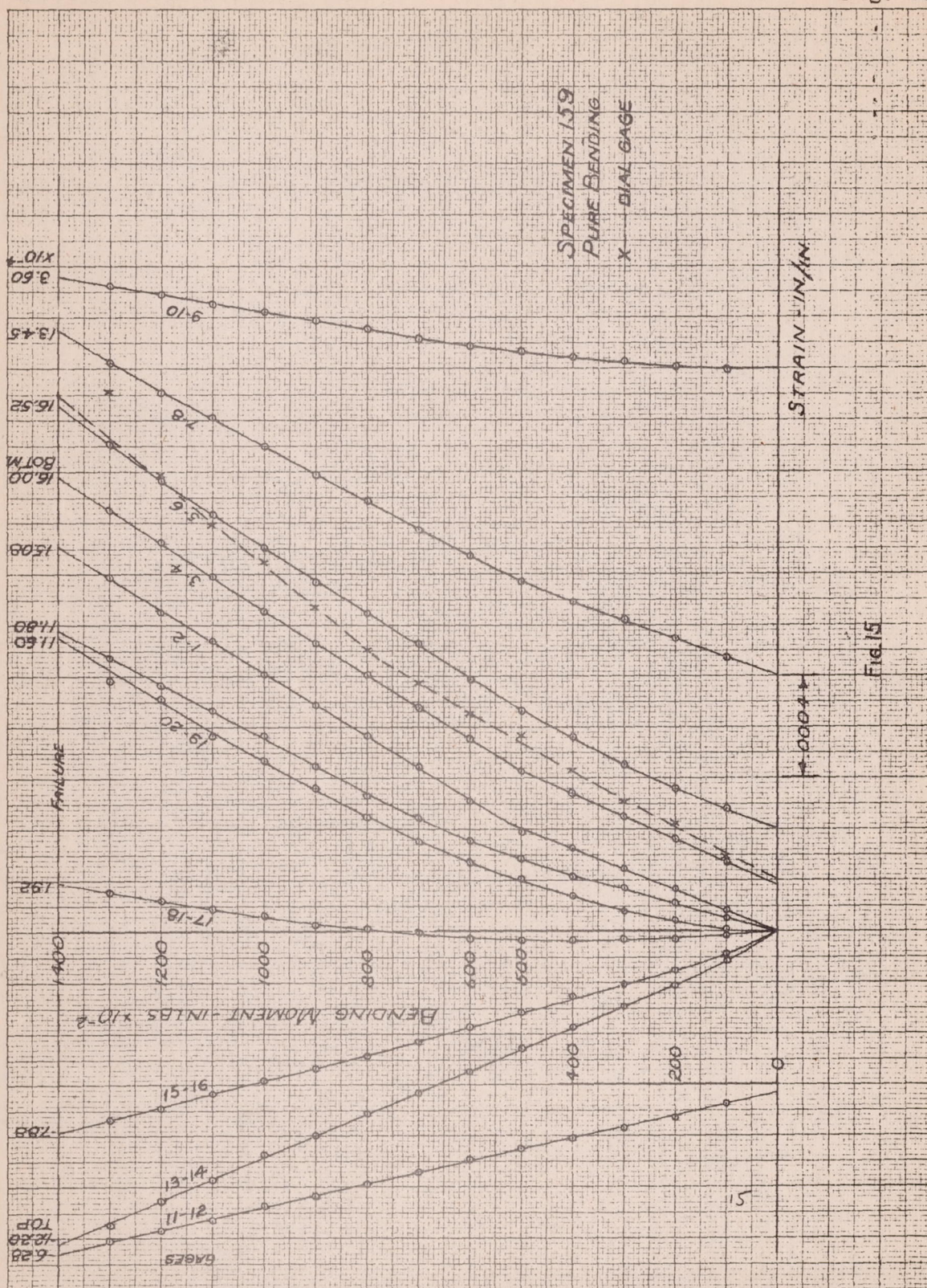




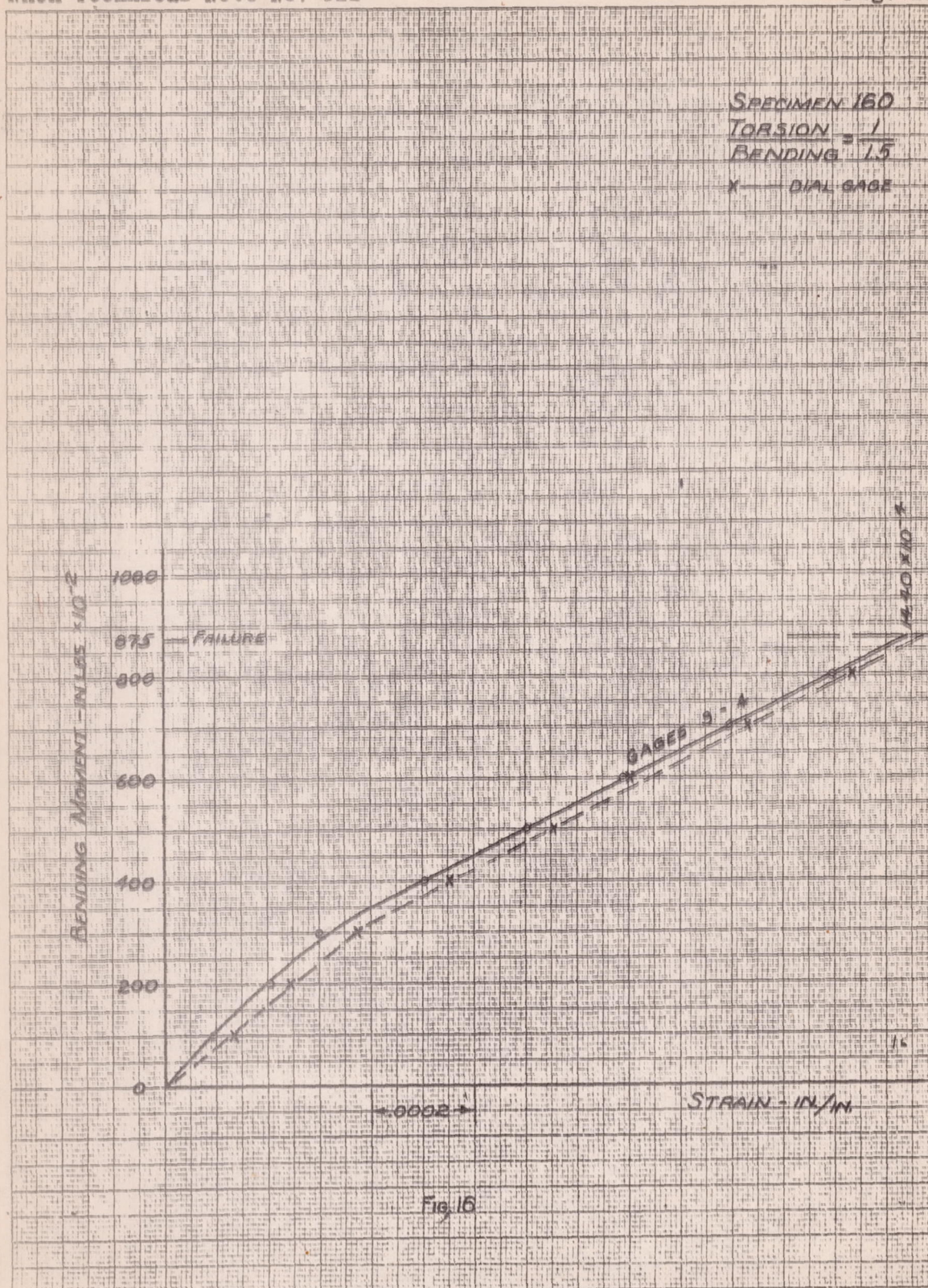




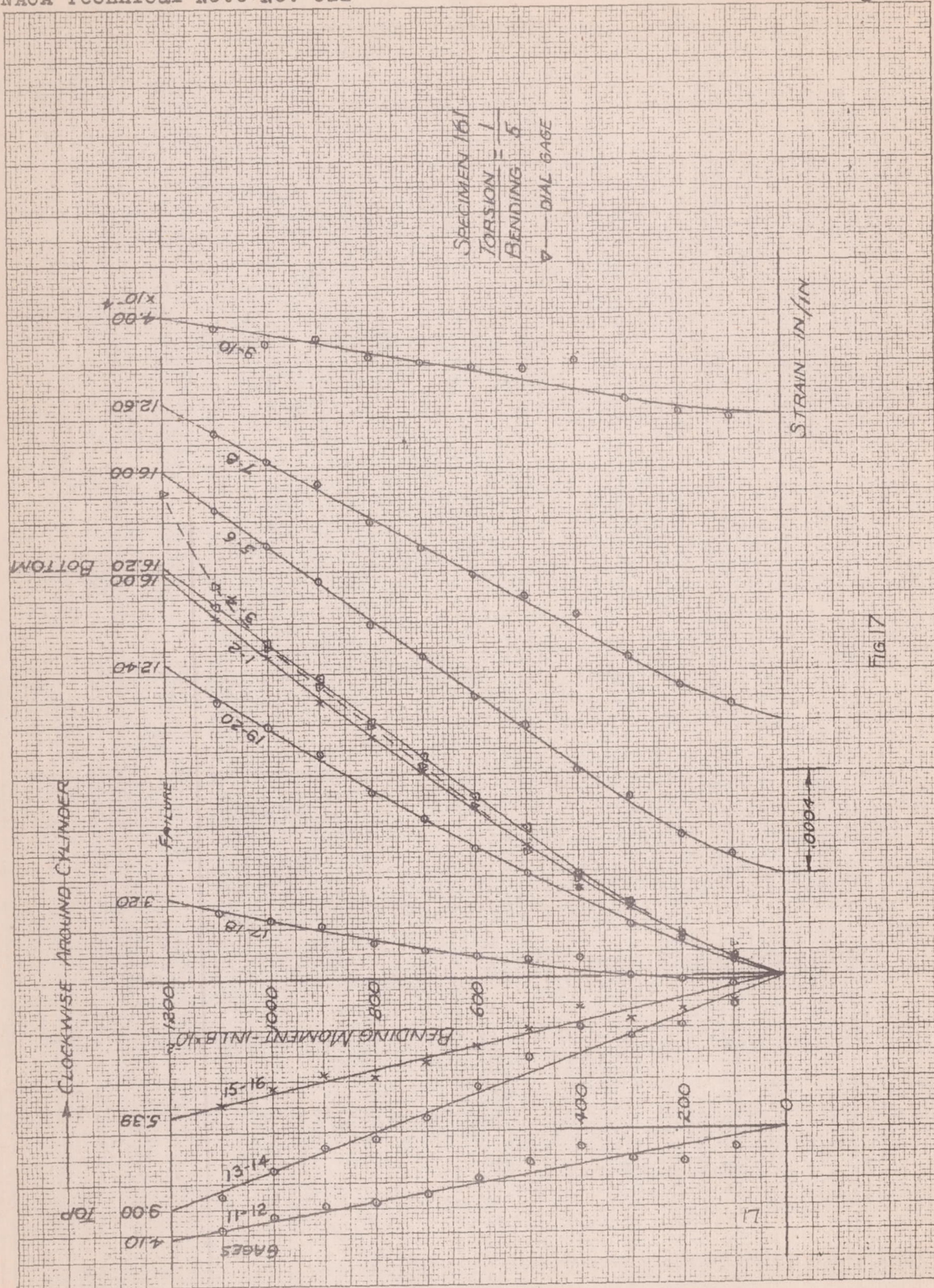




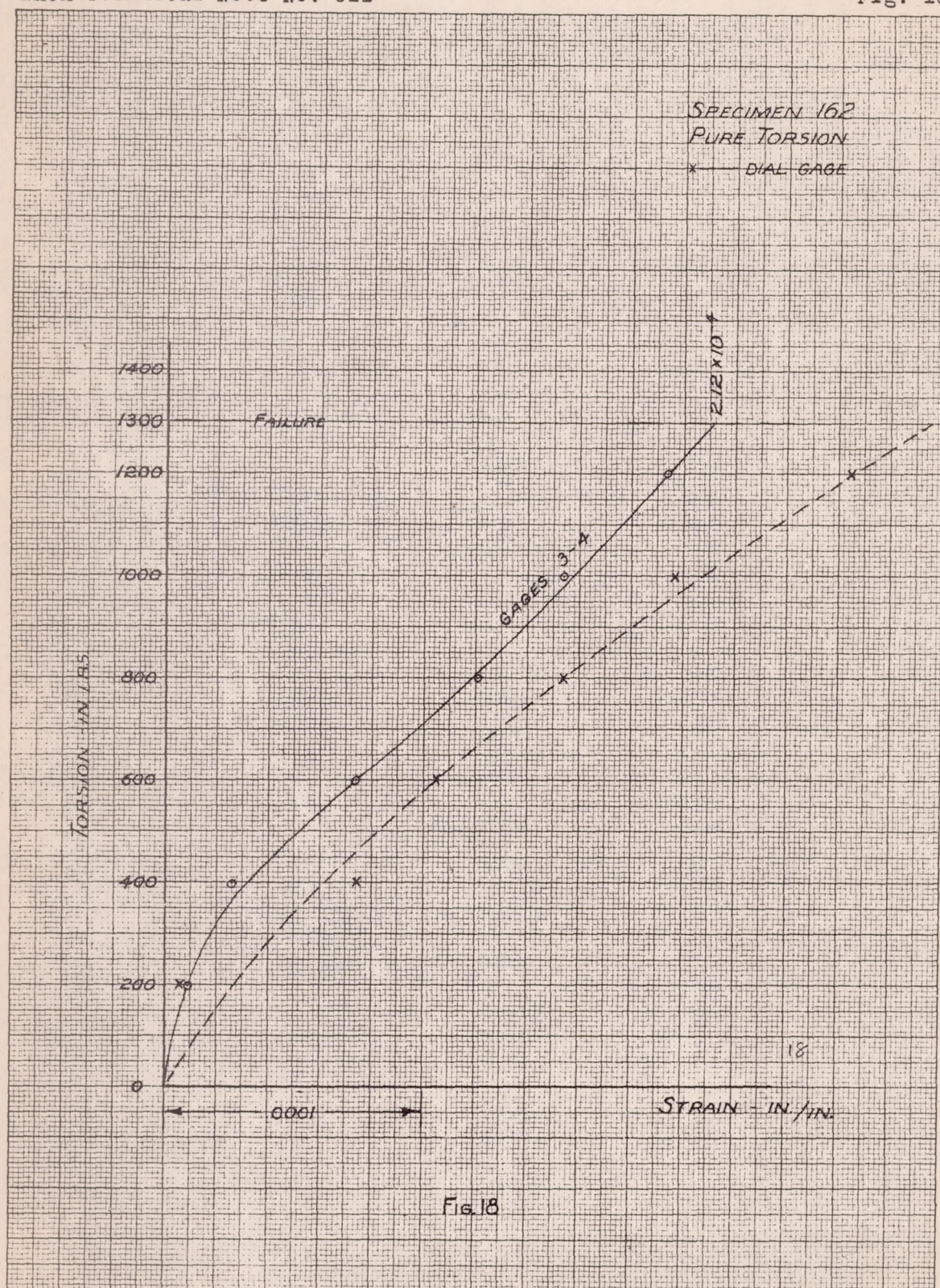




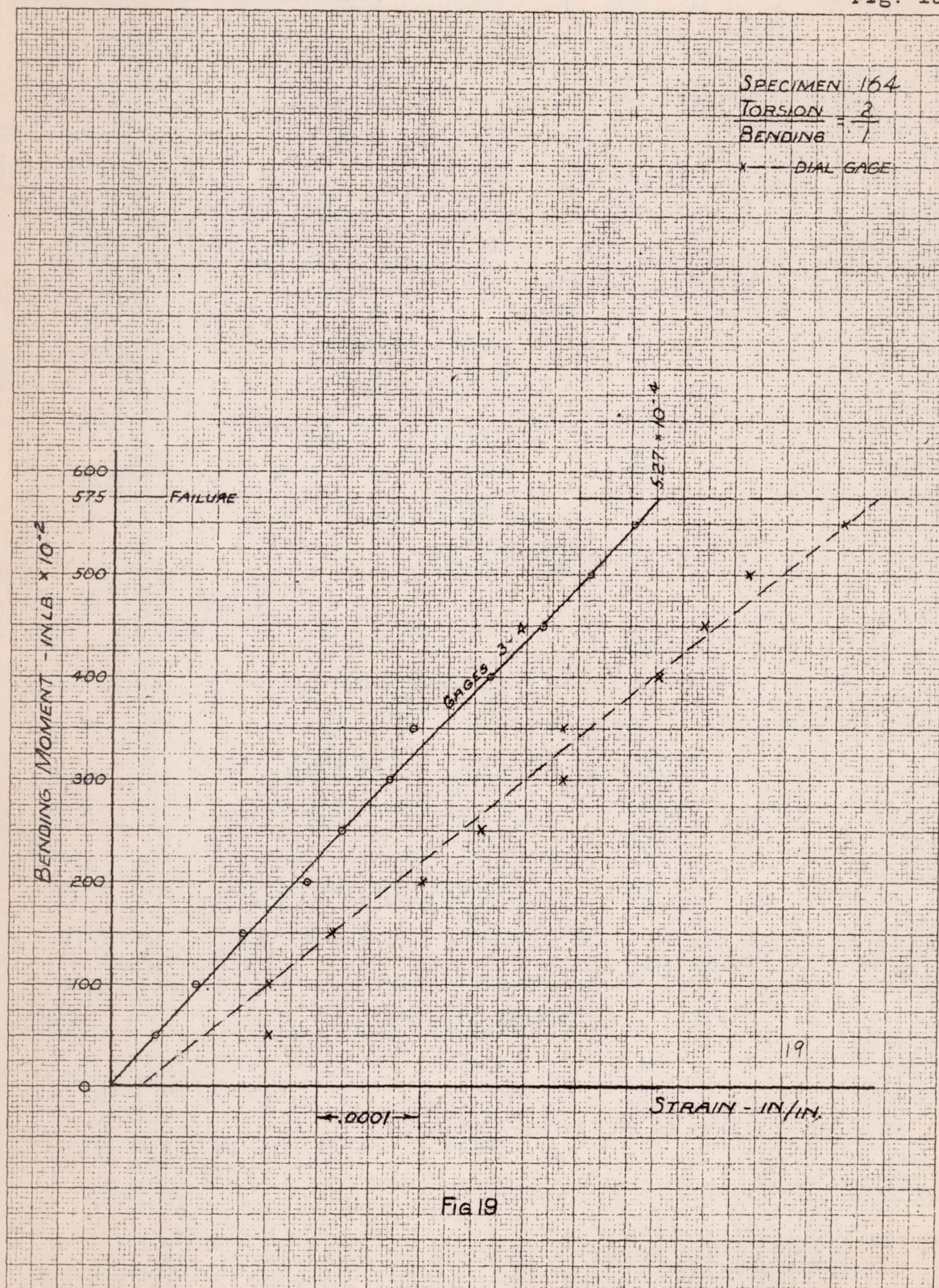




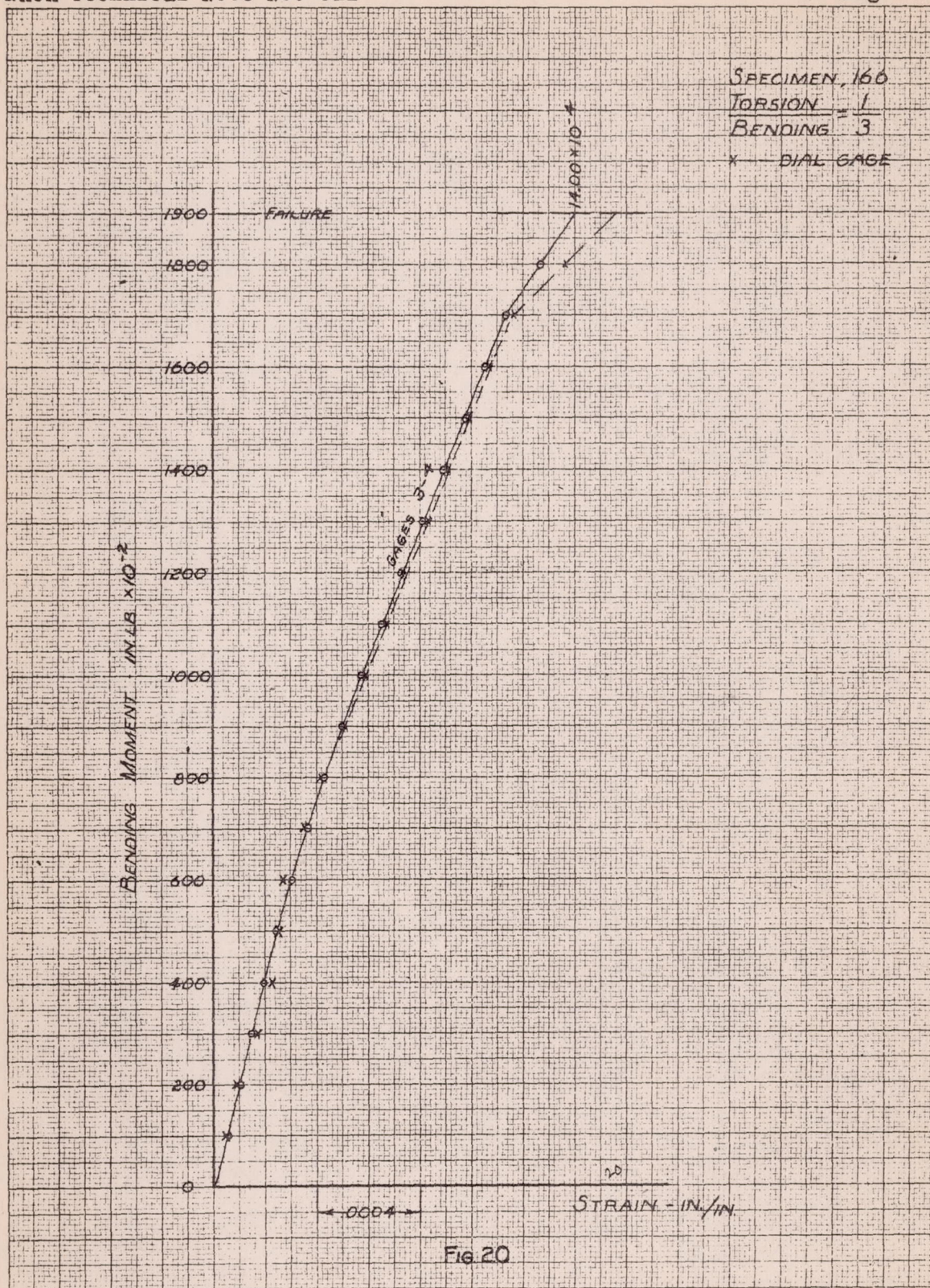




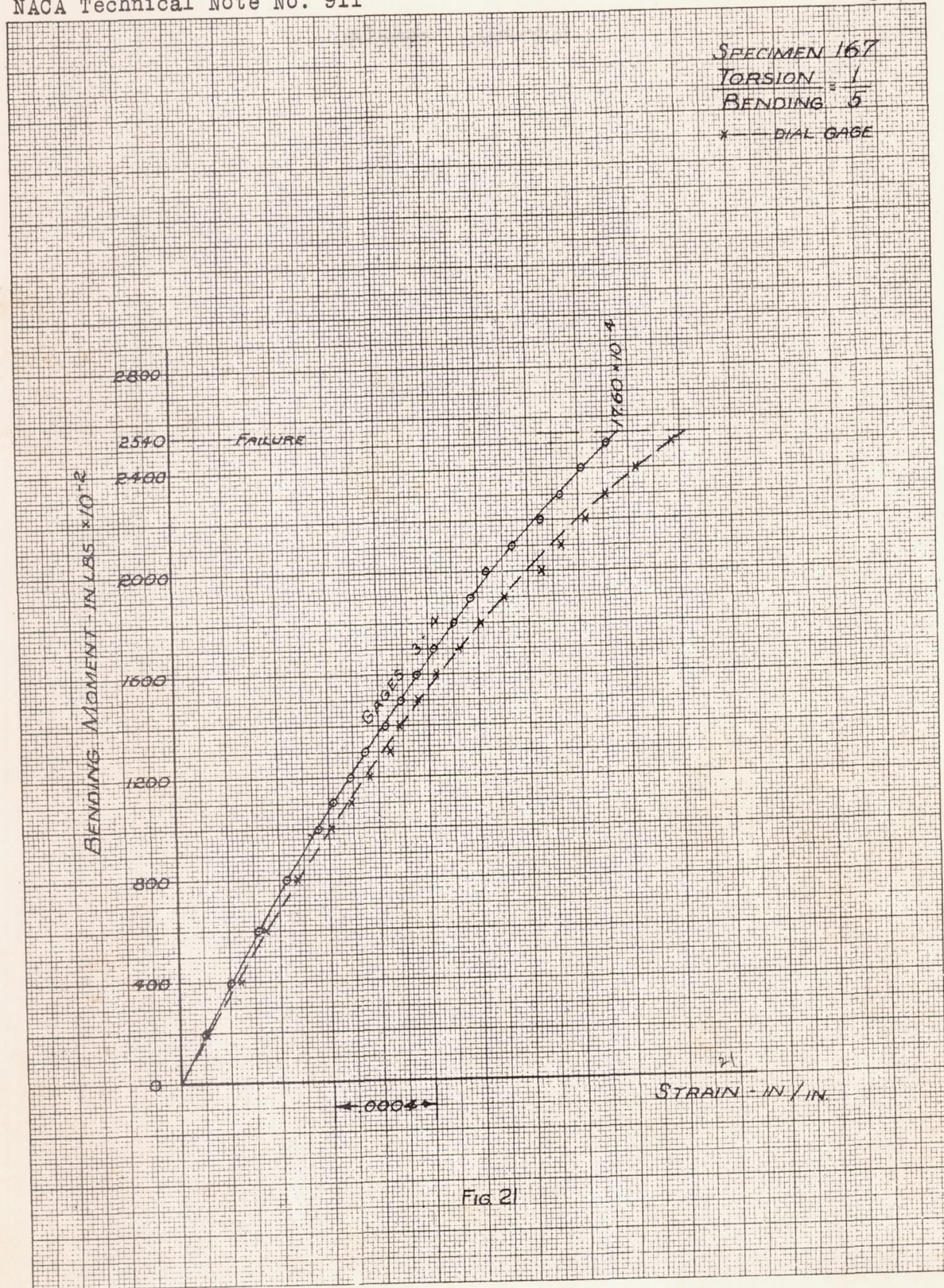














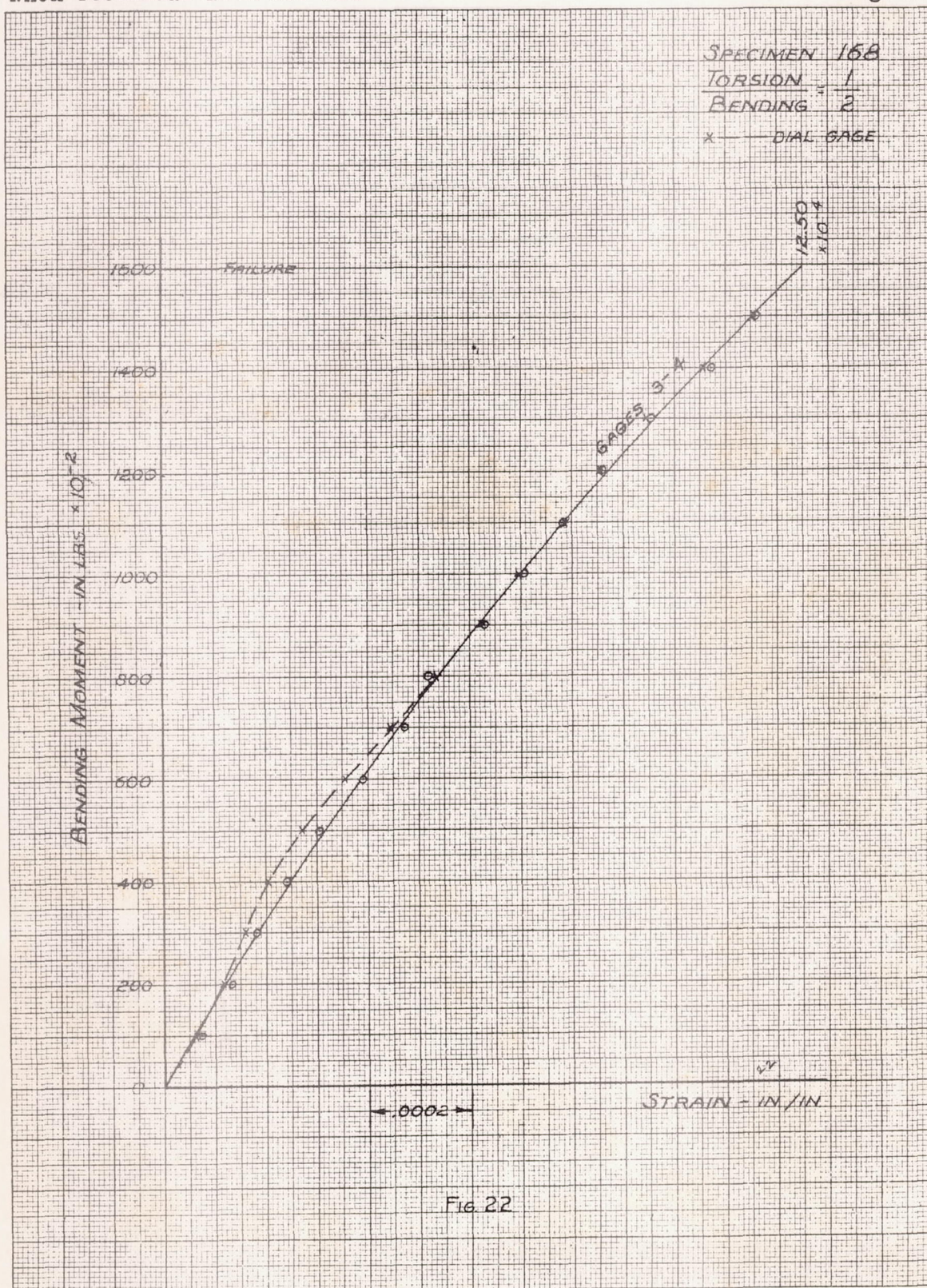
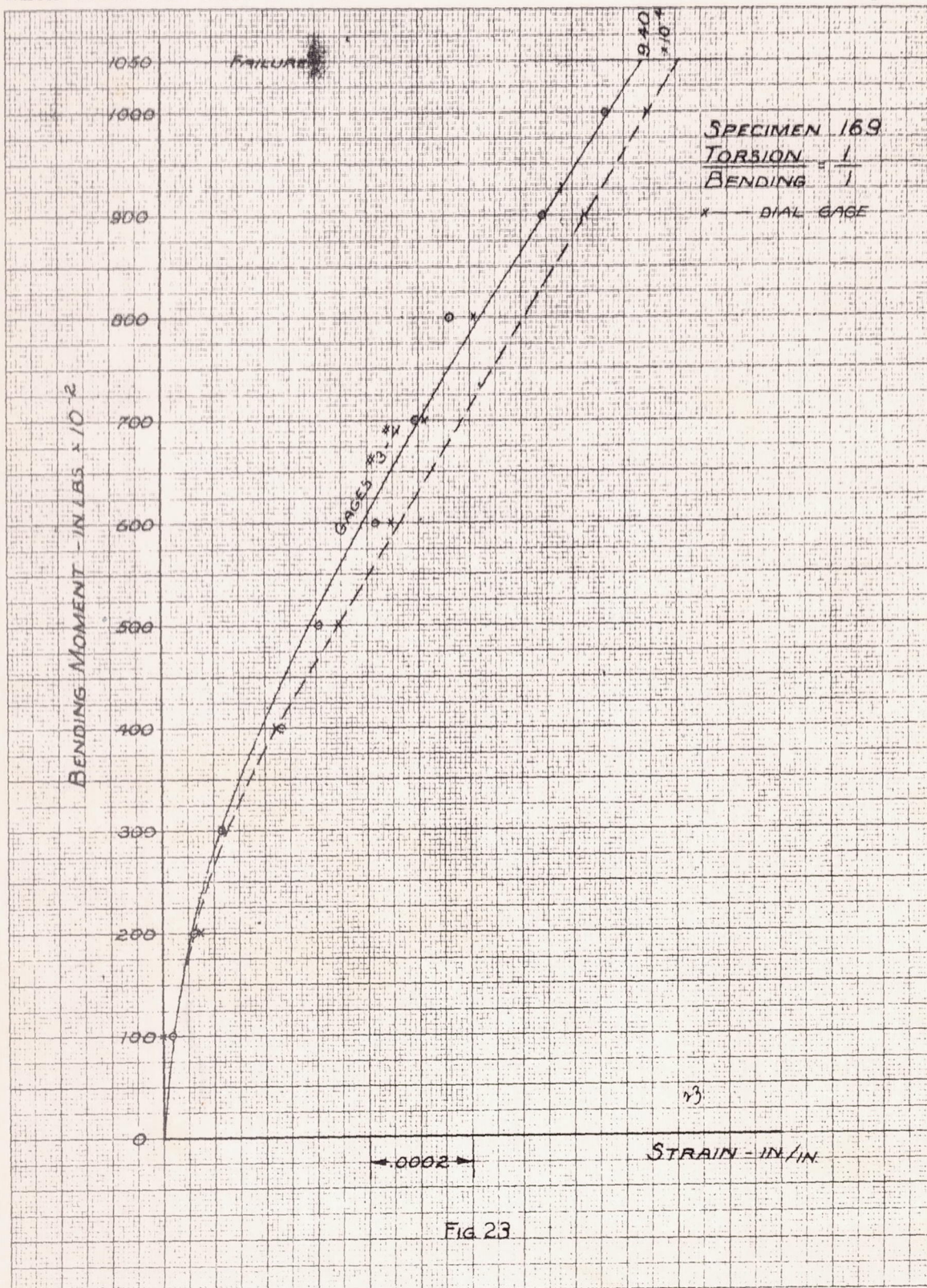


FIG 22







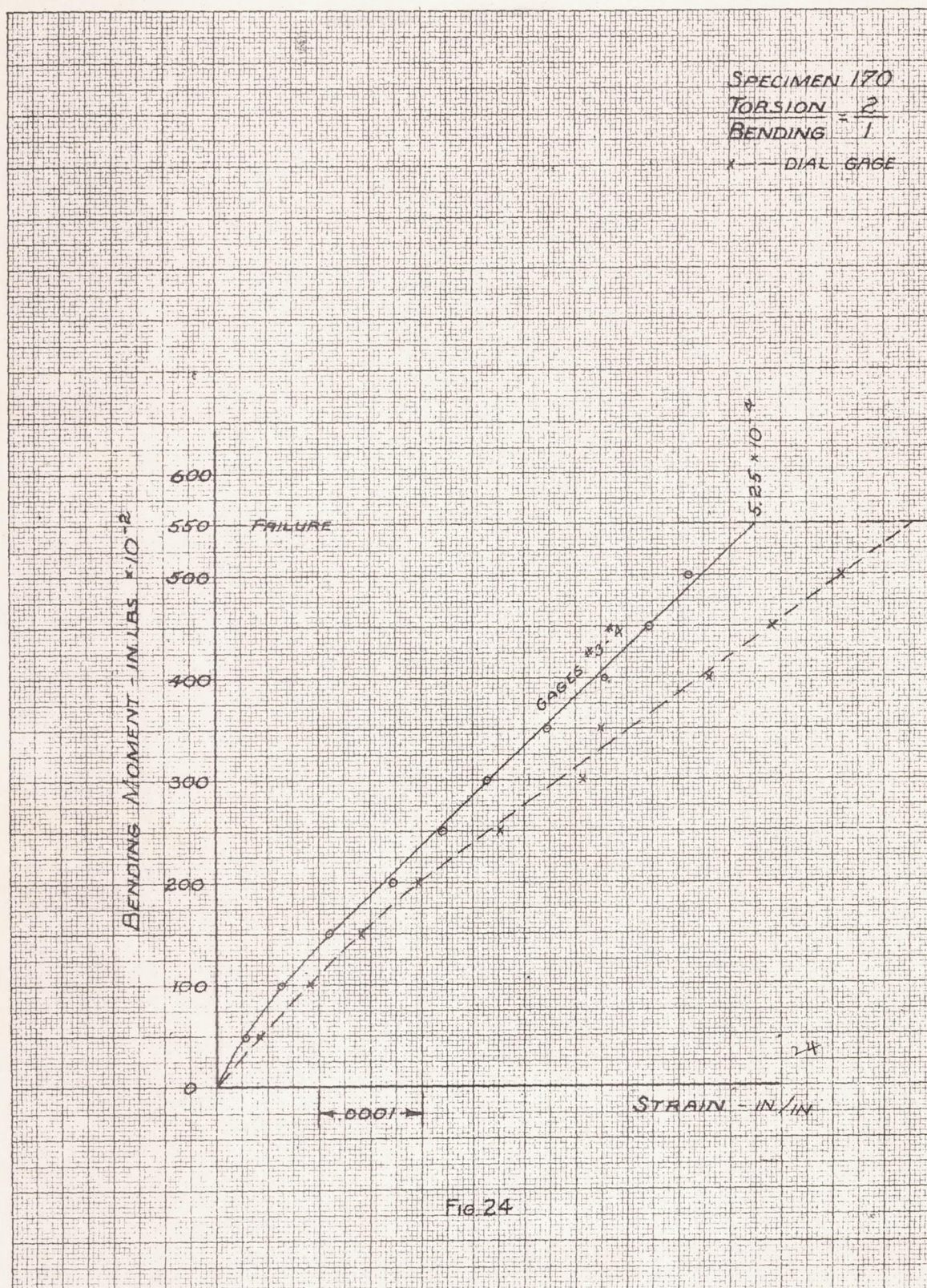


Fig. 24



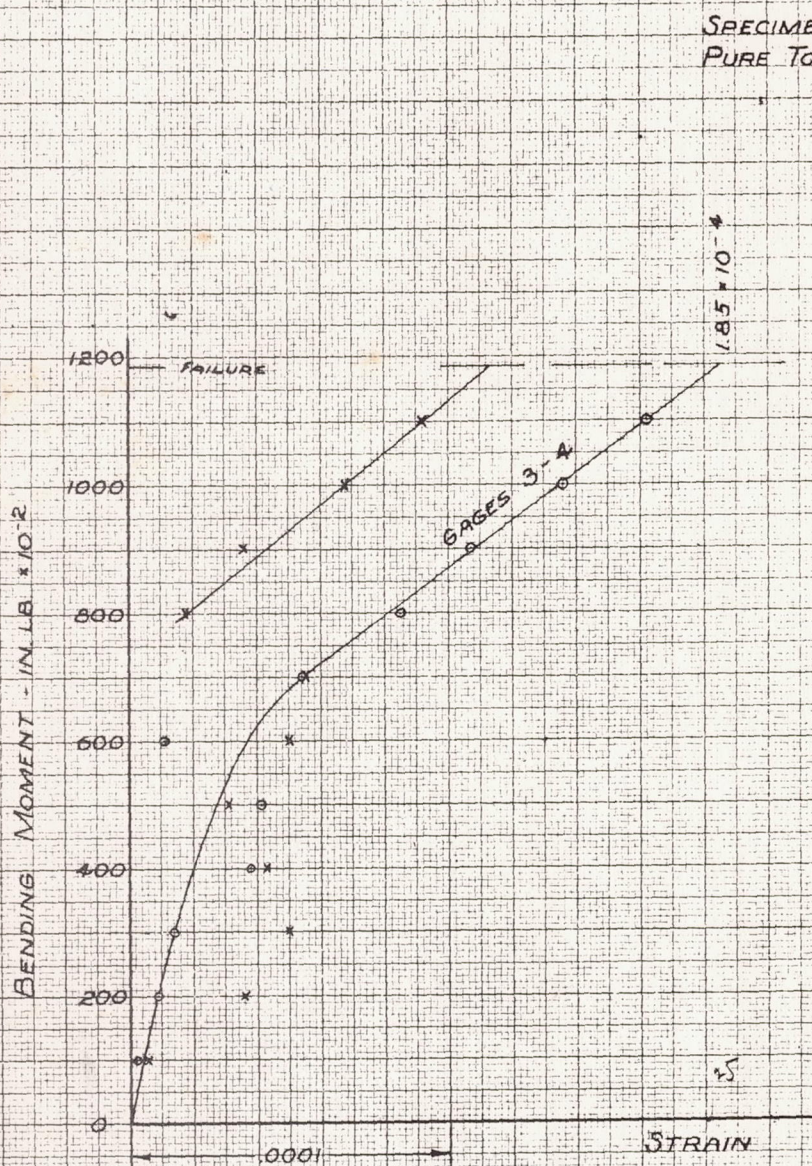
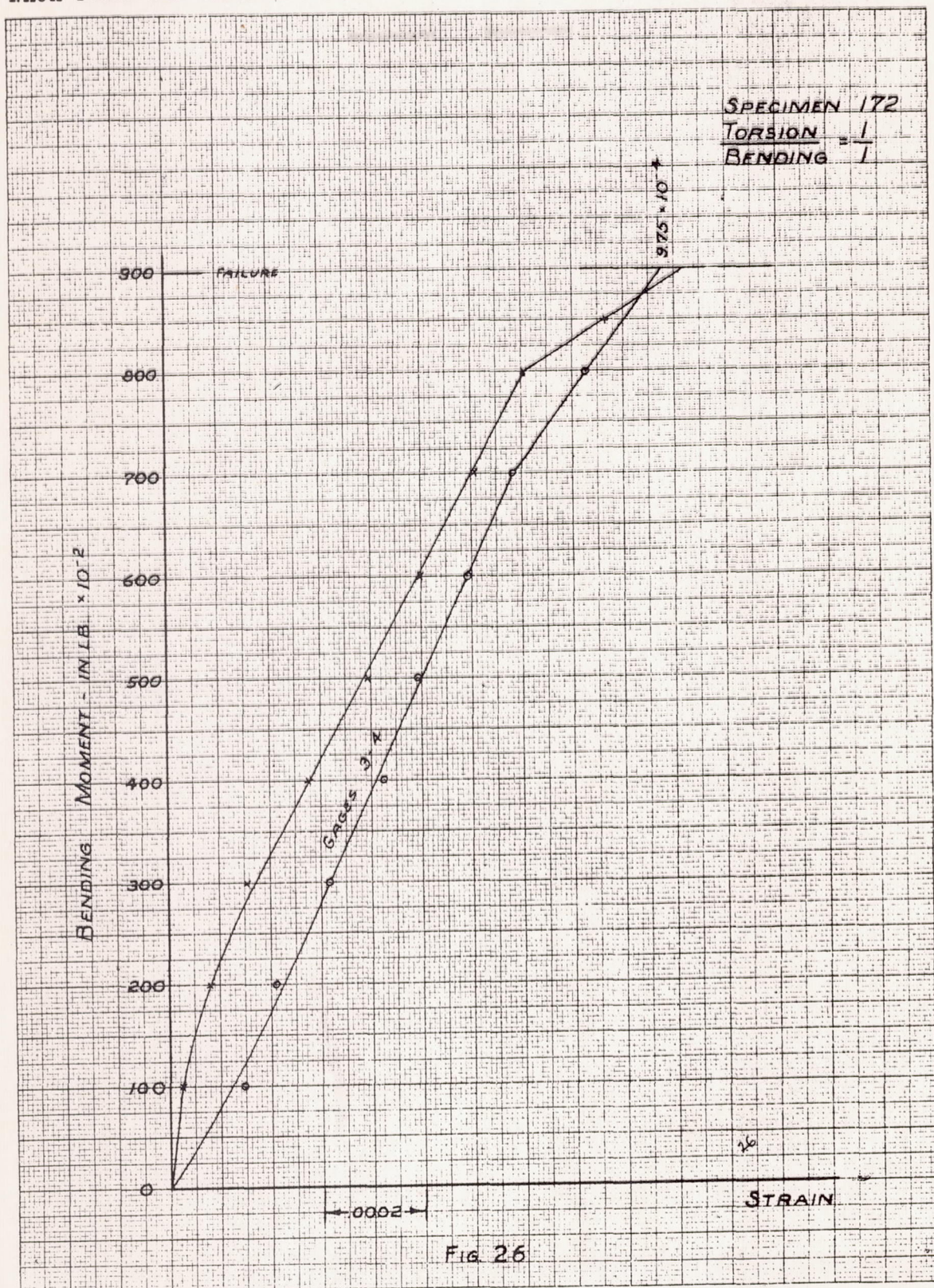
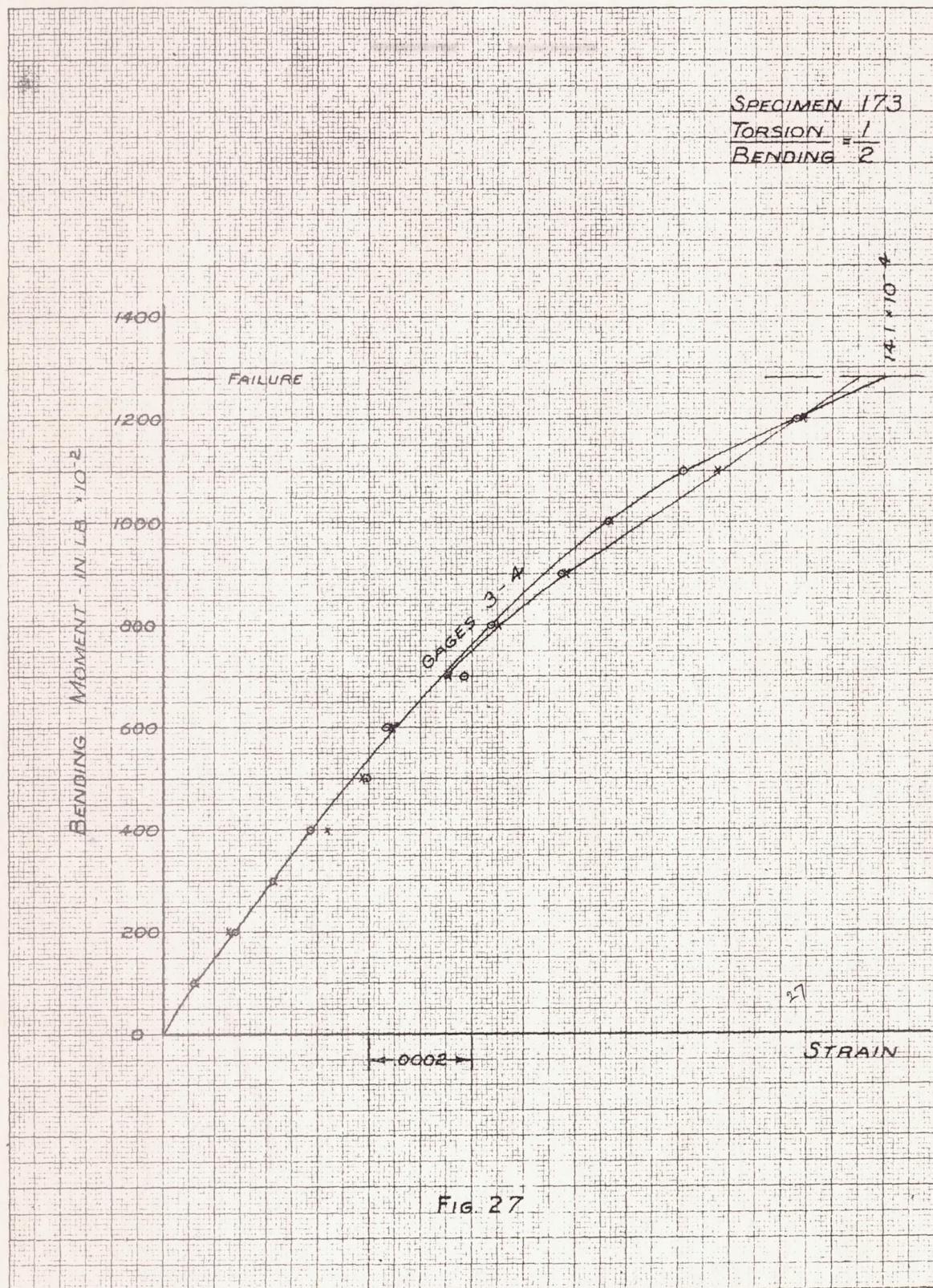


Fig. 25











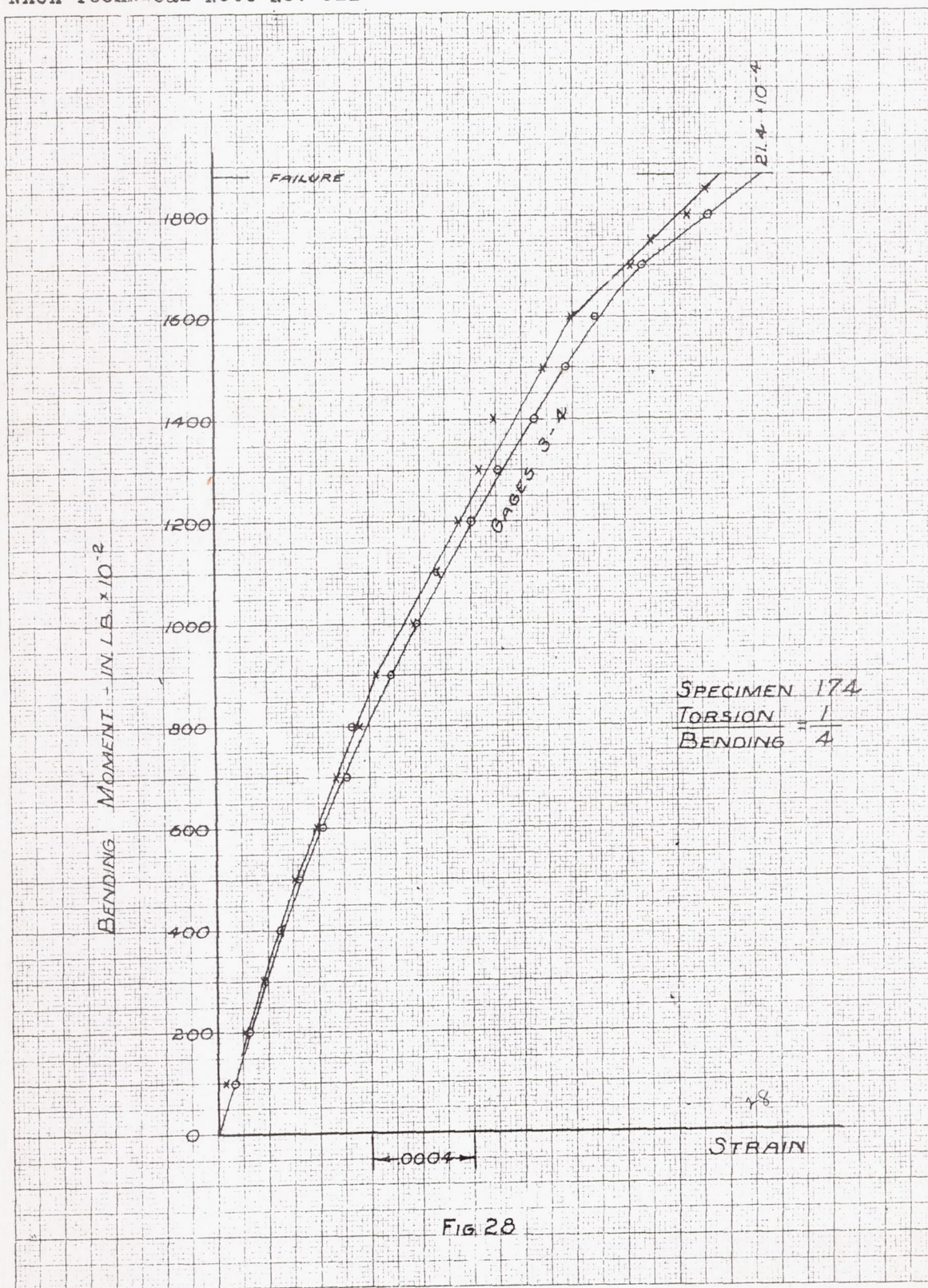


FIG. 28



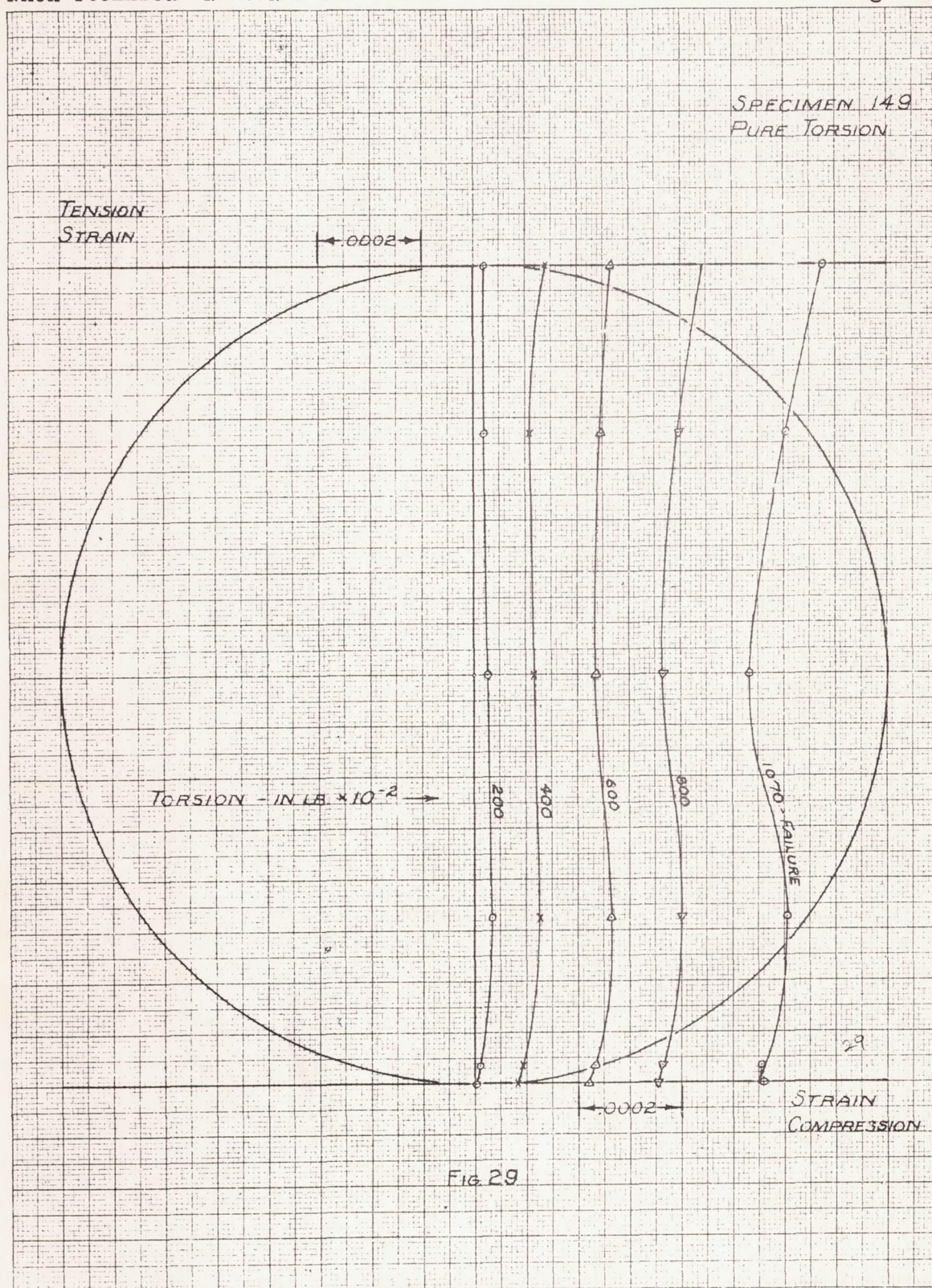
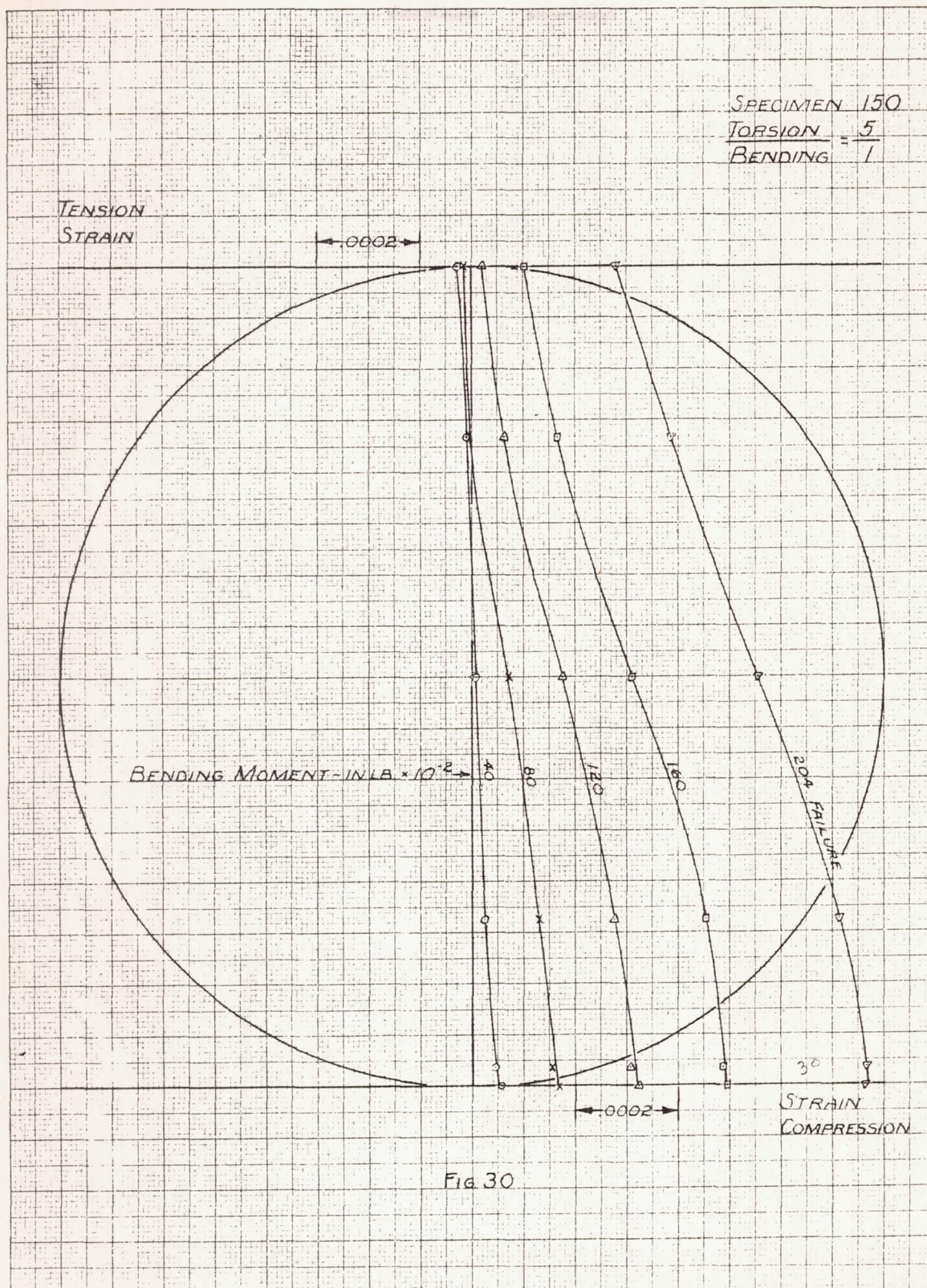


FIG 29







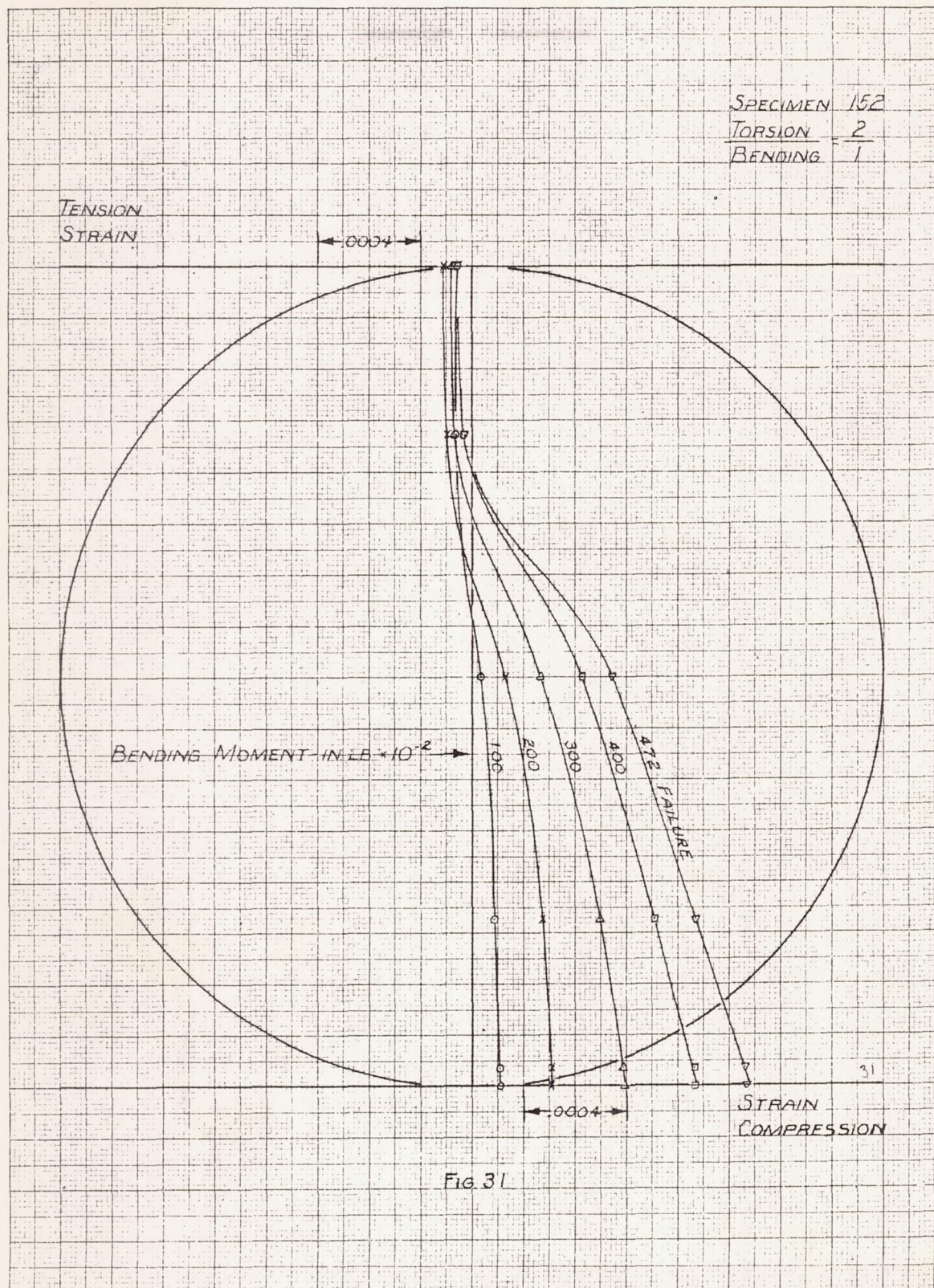
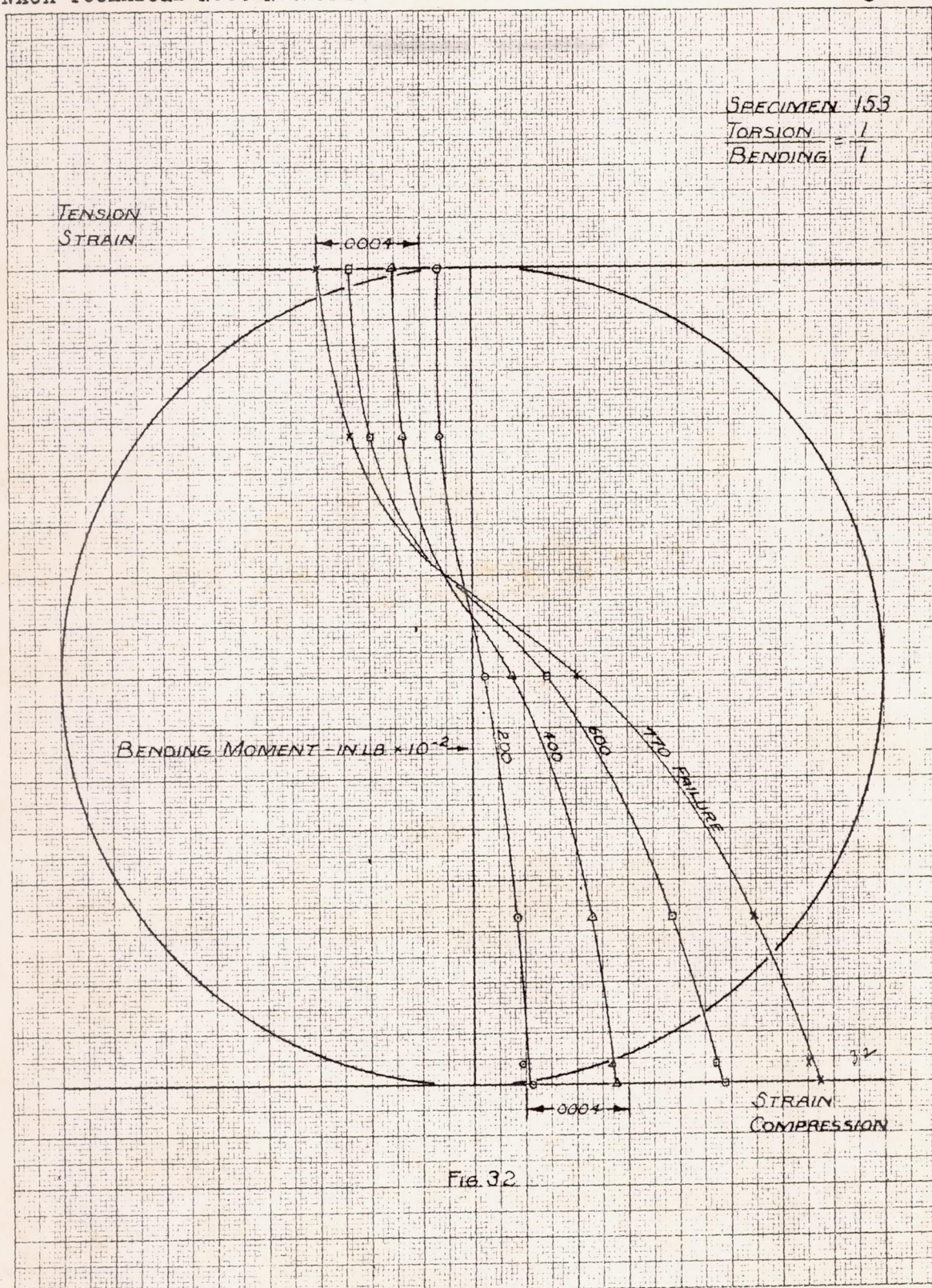
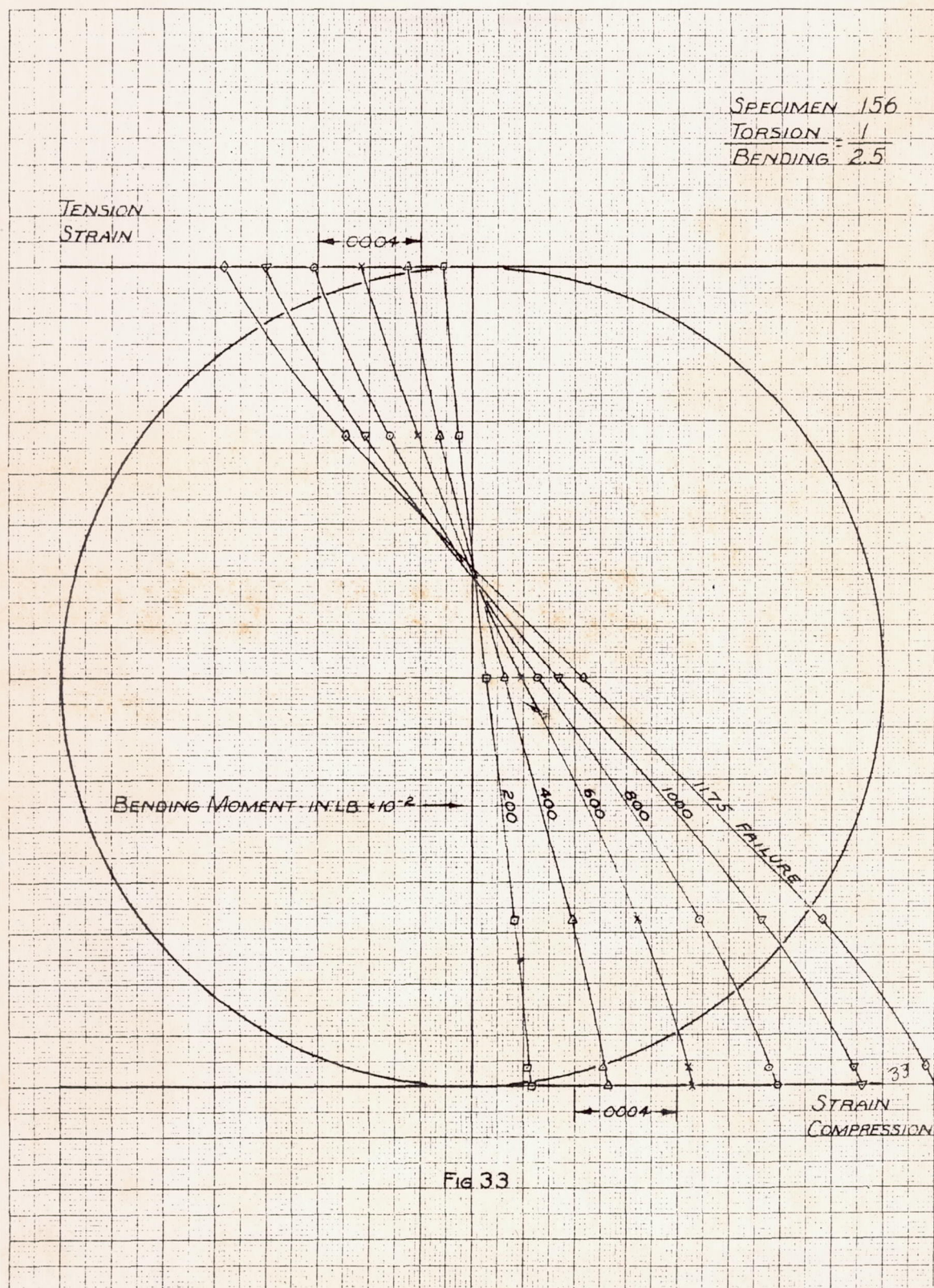


Fig. 31

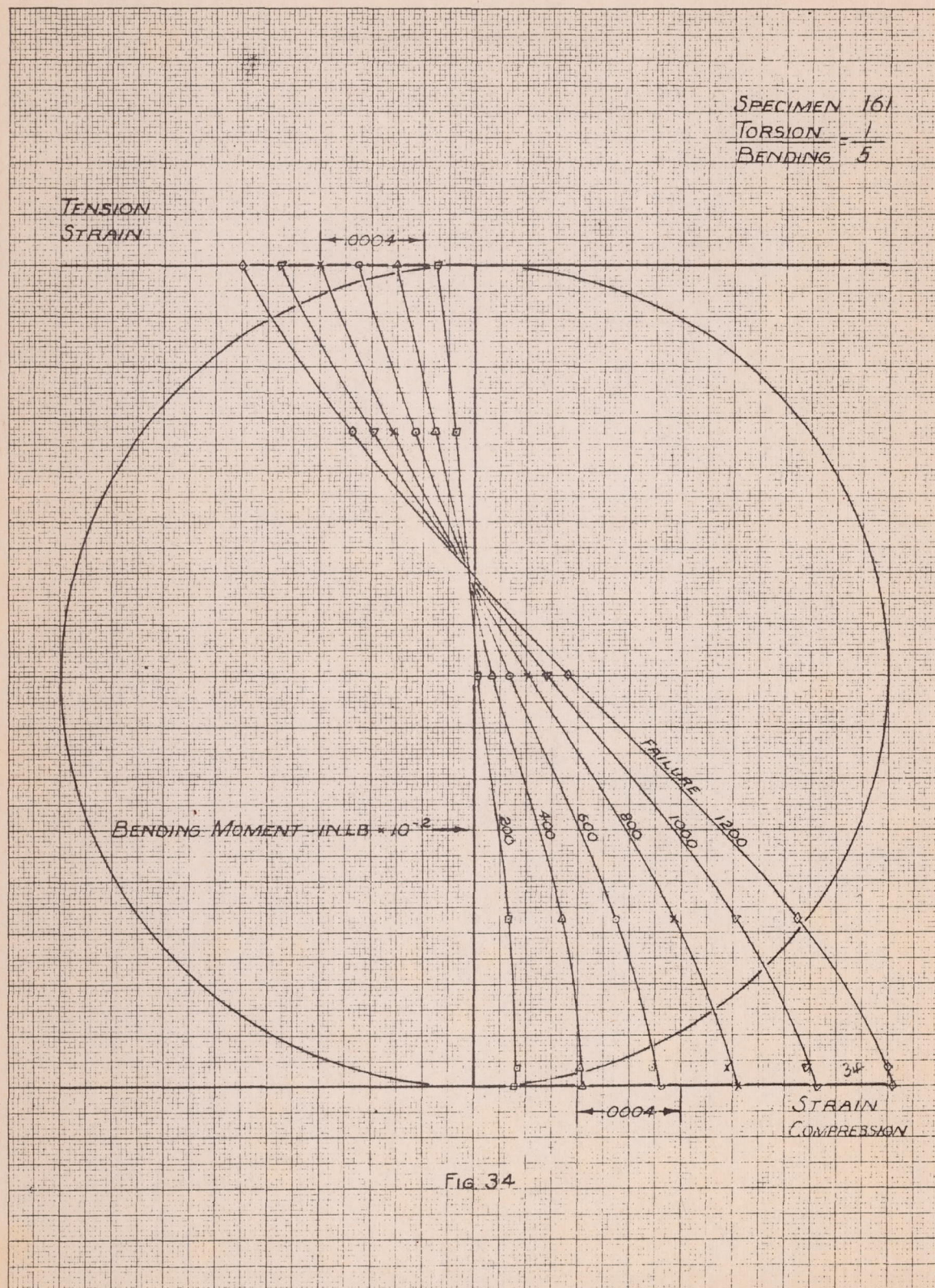




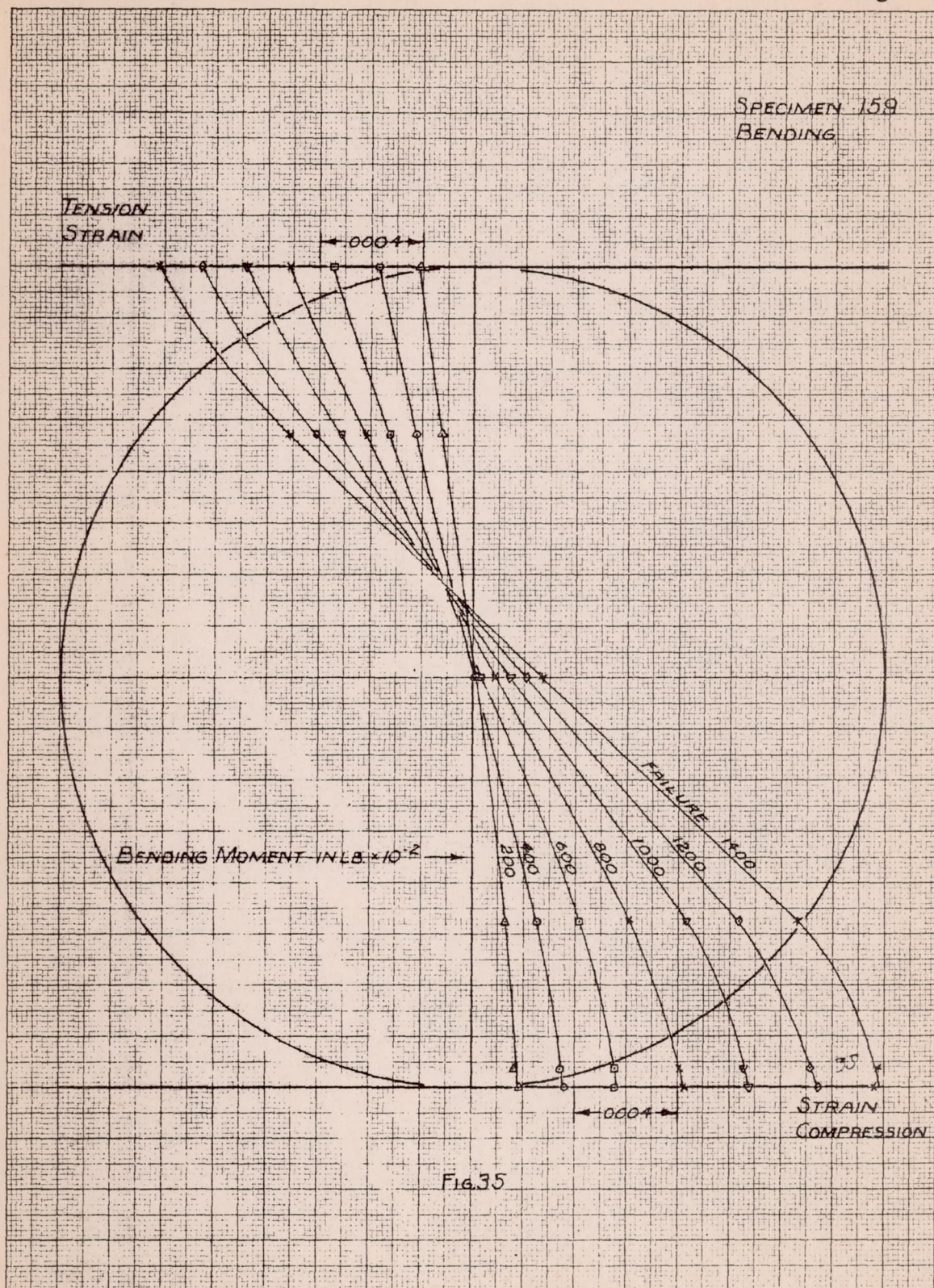














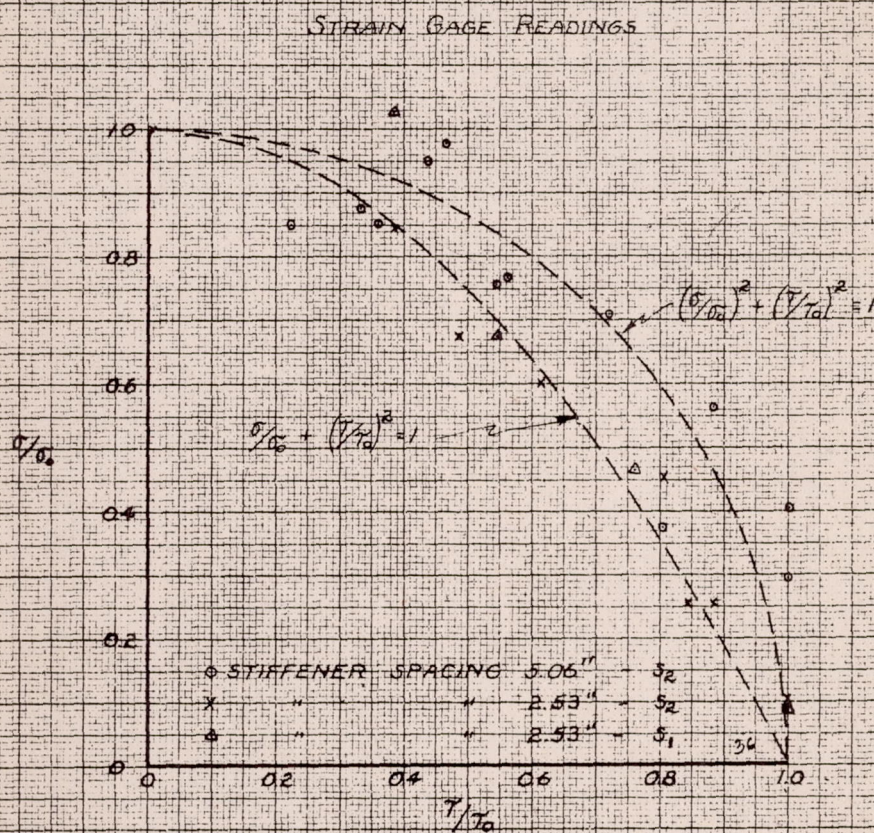


Fig. 36



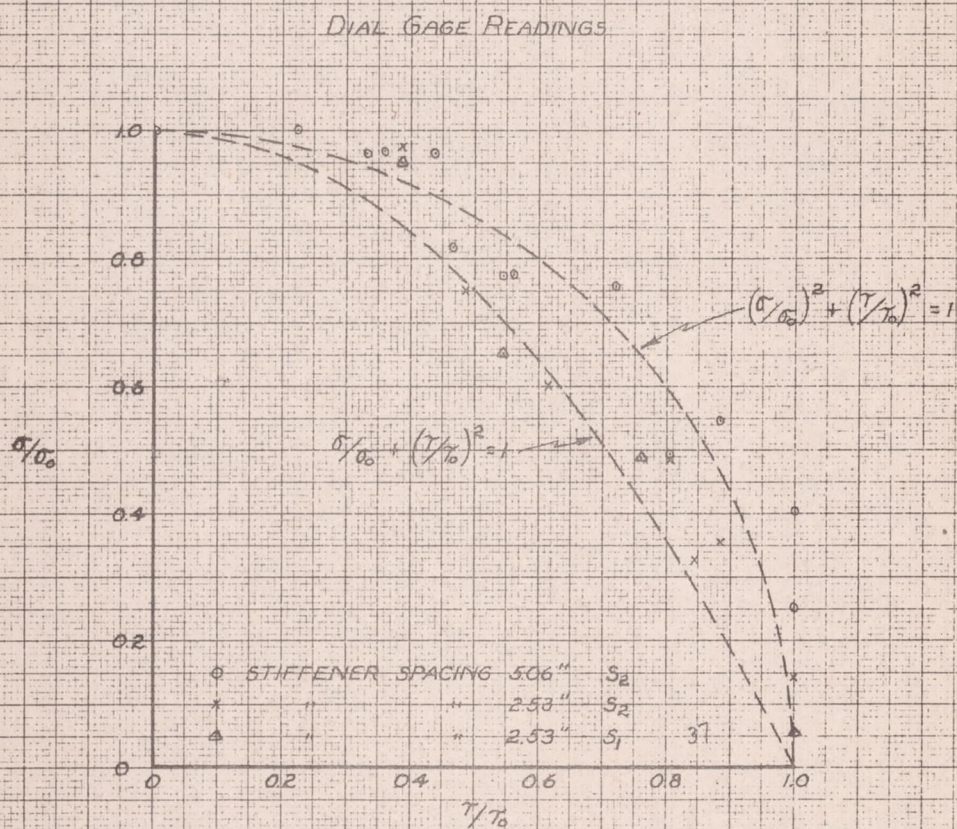


FIG. 37



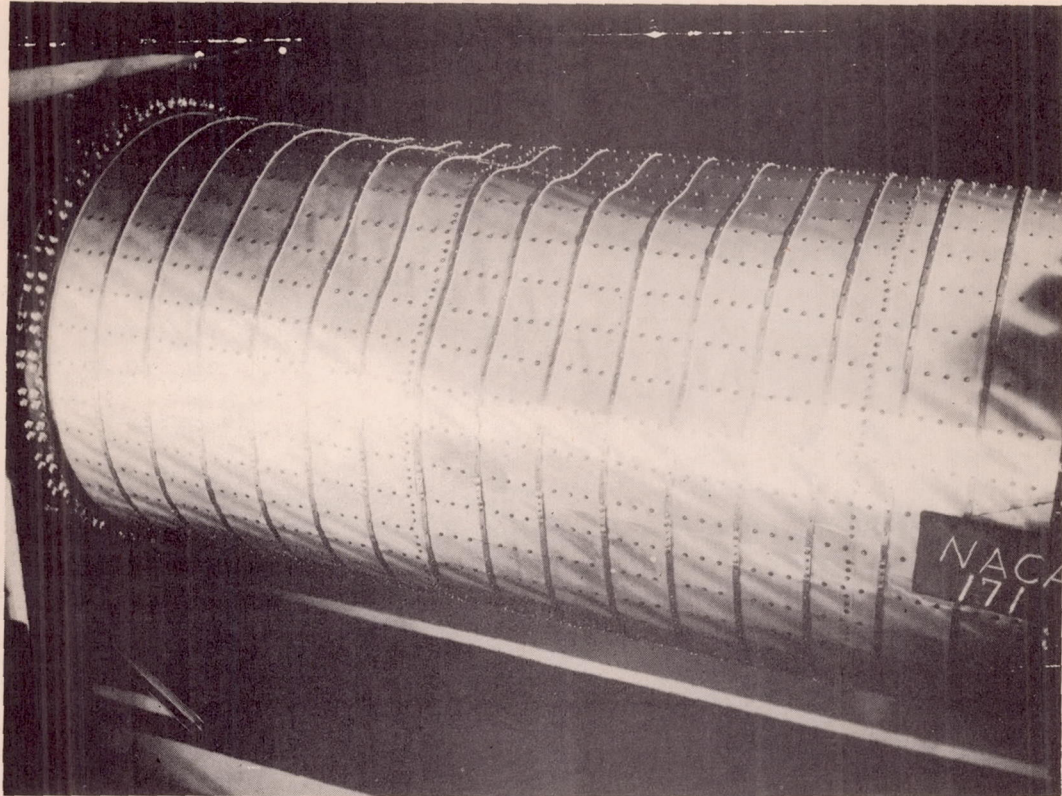


Figure 38.- Pure torsion.

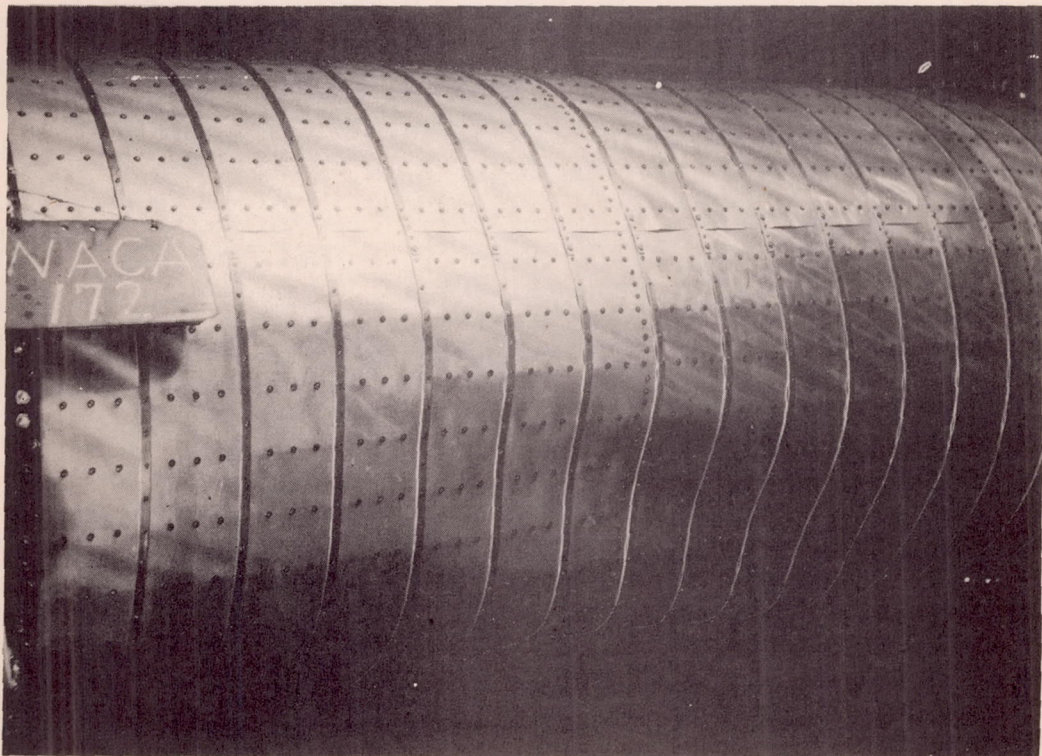


Figure 39.- Combined bending and torsion.



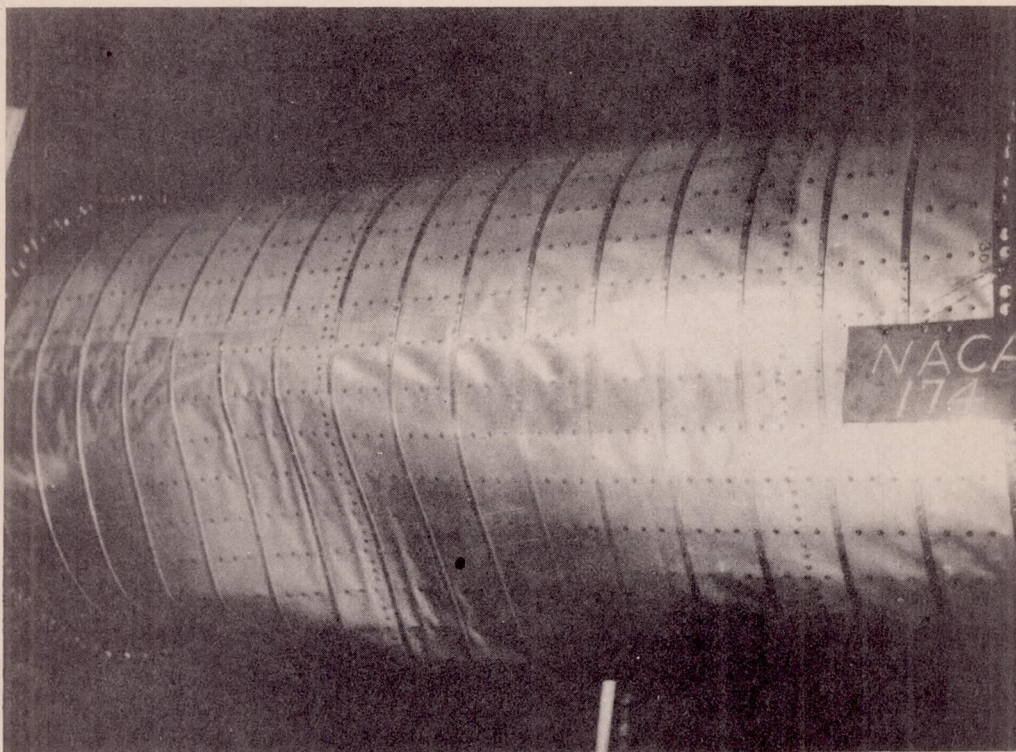


Figure 40.- Combined bending and torsion.

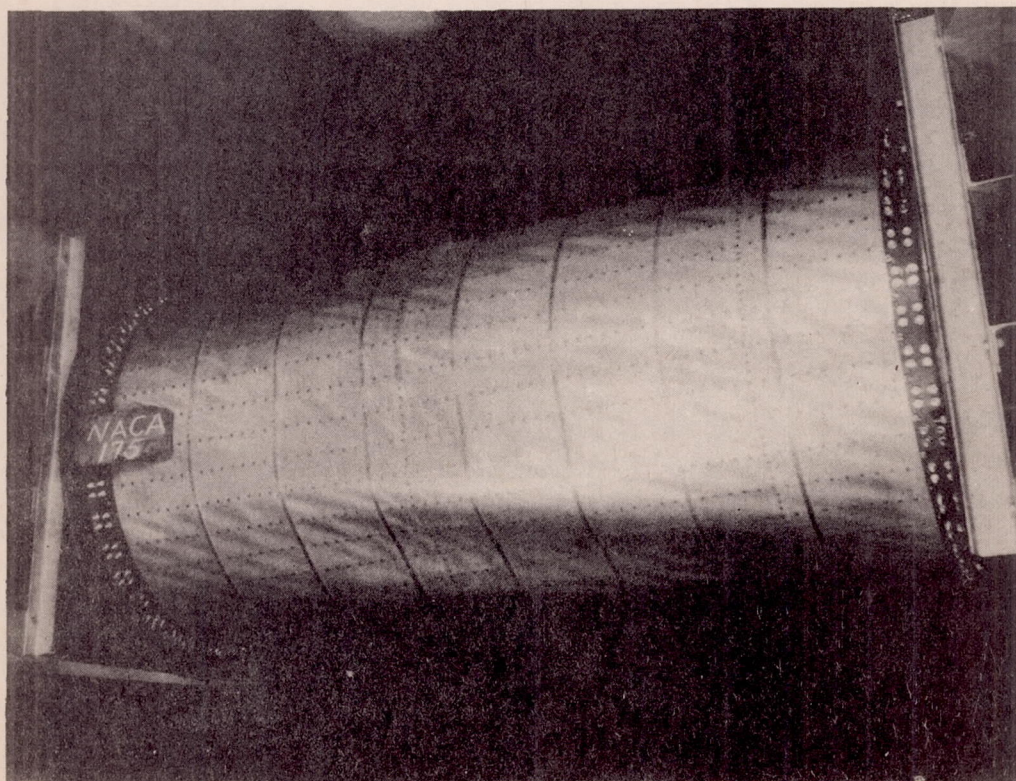


Figure 41.- Pure torsion.



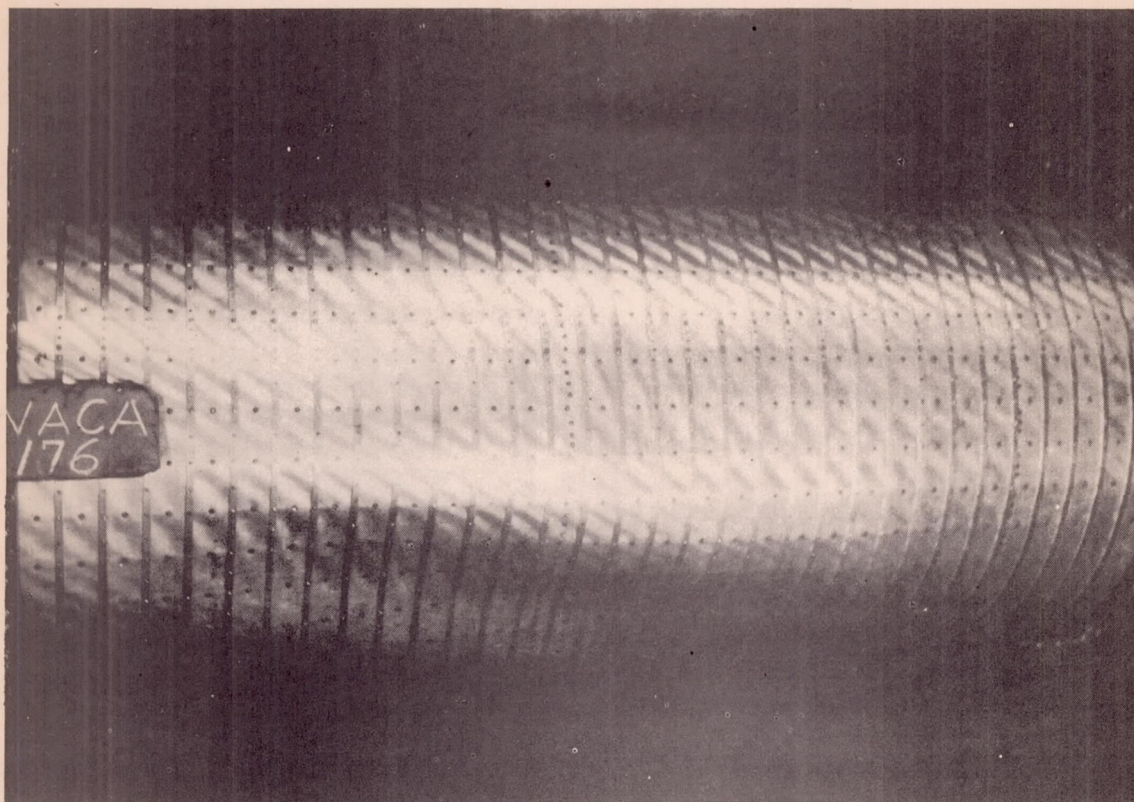


Figure 42.- Pure torsion.

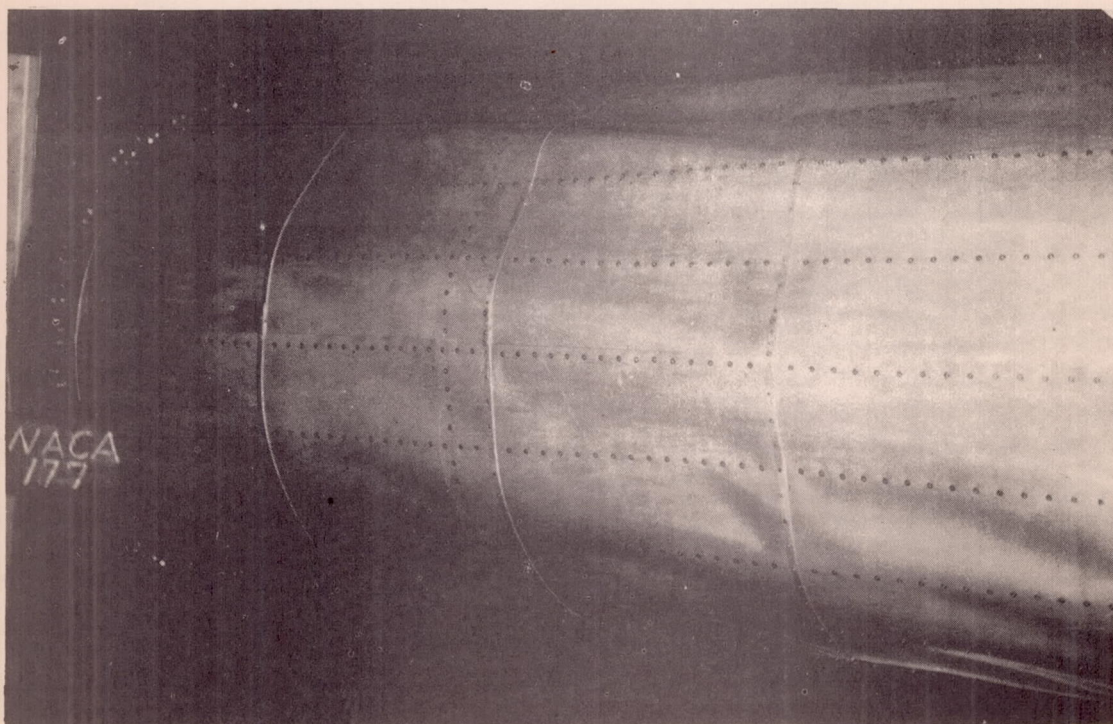


Figure 43.- Pure torsion.